The heat capacity of dense quark-matter phases vs neutron star observational constraint[1]

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[1] **Phys. Rev. D 103 (2021) 123013**.

Composition of Matter in Neutron Stars

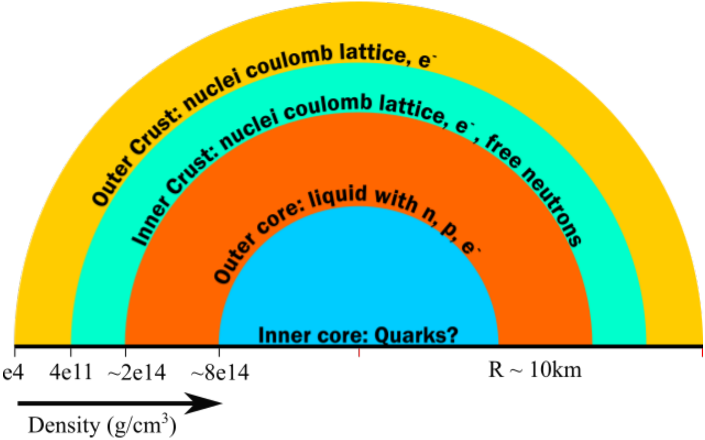

Figure 1: Above we see the representation of a Neutron Star structure. They are thought to consist of a thin crust, which constitutes a few percent of the compact star's mass, and is around $\approx 1km$ and a massive core.

Quark Matter inside Neutron Stars

- Meanwhile, we rely on Neutron Stars (NSs) theoretical studies, using non-perturbative methods through effective models.
- Using the corresponding EoS of different cold-dense phases we calculate different NS's attributes as mass, radius, heat capacity, tidal deformation, etc. for comparison with observation values.
- Our objective is to investigate if a neutron star core formed by a quark-matter phase can be compatible with the observable NSs thermal properties.

Neutron Star thermal history

- Without an active core the NS internal temperature reduces over time and this process can be divided in roughly two stages:

1. When a NS is born its temperature is around $10^{11}K$, then its predominant cooling mechanism during at least a thousand years is neutrino emission from the core.
2. When the temperature decays up to $10^{8}K$, then photon emission from the surface dominates.

- As a first approximation, the thermal history takes place through the following equation:

$$-\frac{dE}{dt} = -C_V \frac{dT}{dt} = L_\nu + L_\gamma - H \qquad (1)$$

- It's clear that the heat capacity C_V plays a fundamental role in the thermal evolution of NSs.

Neutron Star Thermal History

- Continued observations of the temperatures of accreting NSs in quiescense revealed a lower limit constraint to the heat capacity $C_V \gtrsim 10^{36}(\frac{T}{10^8})\frac{erg}{K}$[2].
- To determine if quark matter can be a candidate for NS interior we will investigate if its C_V satisfies the observational constraint.
- With this goal, we will consider the CFL phase, which is favorable at very high densities and the MDCDW phase, which is favorable at intermediate densities.

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[2] Cumming, Andrew et al., *Phys. Rev. C*,. 95, p. 025806, 2017.

The QCD Map

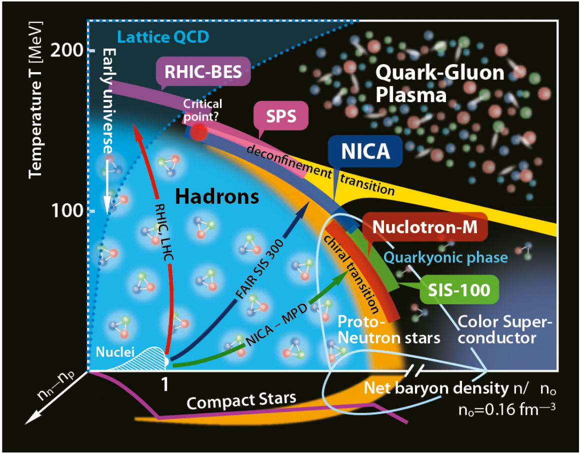

Figure 2: QCD phase diagram from HIC vs. Astrophysics

The Heat Capacity of the CFL phase

From statistical mechanics we know that

$$C_V = -T\left(\frac{d^2\Omega}{dT^2}\right)_V = -2\beta^2\left(\frac{d\Omega}{d\beta}\right)_V + \beta^3\left(\frac{d^2\Omega}{d\beta^2}\right)_V, \qquad (2)$$

Ω is the Grand Canonical potential which comes from the Grand partition function. where $\beta = 1/T$, with T the absolute temperature. For the CFL phase, in the low temperature limit, we found

$$C_V^{CFL} \simeq \frac{(\mu^2+\Delta^2)T}{3\pi^2}e^{-\sqrt{\mu^2+\Delta^2}/T} + \frac{(\mu^2+4\Delta^2)T}{3\pi^2}e^{-\sqrt{\mu^2+\Delta^2}/T} \qquad (3)$$

The Heat Capacity of the CFL phase

- Since the C_V of this phase is exponentially damped, it can be shown that it does not satisfy the lower-limit constraint on C_V. Moreover, because it is also electrically neutral without the necessity to include electrons, it cannot be realized inside the core of NS.
- For any matter component that presents either superconductivity or superfluidity we found the same behavior, they are all exponentially damped.
- If quark matter were to appear in the core solely via the color superconducting CFL/MCFL phases, then they can't be a constituent part of the core.

The MDCDW phase

- We model cold and dense matter with a Nambu–Jona-Lasinio-QED Lagrangian density,

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu\right)\psi + G\left((\bar{\psi}\psi)^2 + (\bar{\psi}\boldsymbol{\tau}\gamma_5\psi)^2\right) \quad (4)$$

- with $Q = (e_u, e_d) = (\frac{2}{3}e, \frac{-1}{3}e)$, $\psi^T = (u, d)$; μ the baryon chemical potential; and G the four-fermion coupling. The electromagnetic potential A_μ is formed by the background $\bar{A}_\mu = (0, 0, Bx, 0)$, that corresponds to a constant and uniform magnetic field B in the z direction, plus the fluctuation field.
- The presence of B favor the formation of DCDW condensate[3], $\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta e^{ibz}$ with magnitude Δ and modulation vector $\boldsymbol{b} = (0, 0, b)$.

[3] Ferrer, E.J. and de la Incera, V., *Physics Letters B*,. 769, pp. 208–212, 2017.

The MDCDW phase

- Given that the temperature-dependent thermodynamic potential of the MDCDW phase

$$\Omega_\beta^{MDCDW} = -\sum_{f=u,d} \frac{|e_f B| N_c}{(2\pi)^2 \beta} \int_{-\infty}^{\infty} dp \sum_{l\xi\epsilon} \ln\left(1 + e^{-\beta(|E^f_{l,\xi,\epsilon} - \mu|)}\right) \tag{5}$$

where N_c is the color number, f denotes the flavor index for quarks u and d, l is the Landau level number, and the energy spectra are given by

$$\begin{aligned} E_{0,\epsilon} &= \epsilon\sqrt{m^2+p^2}+b, \quad \epsilon=\pm, l=0 \\ E^f_{l,\xi,\epsilon} &= \epsilon\left[\left(\xi\sqrt{m^2+p^2}+b\right)^2+2|e_f B| l\right]^{1/2}, \\ & \epsilon=\pm, \xi=\pm, l=1,2,3,... \end{aligned} \tag{6}$$

The Heat Capacity of the MDCDW phase

Substituting Eq. (5) into Eq. (2), in the intermediate density regime we found in the low-temperature limit

$$C_V^{MDCDW} \simeq \frac{eBT}{2} \tag{7}$$

Immediately, we notice that the heat capacity isn't so damped as in the CFL phase.

- Now we will calculate the value of the heat capacity for a Neutron Star with typical values of radius R = 10km and temperature $T = 10^8 K$.

The Heat Capacity of the MDCDW phase

From the model, see Eq. (Eq. 5) we have that the quark number density for the MDCDW phase is given by[4]: $n_q = n_{anom} + n_{ord}$

- Here, $n_{anom} = 3\frac{|eB|b}{2\pi^2}$ is the particle number density coming from the anomalous contribution of the thermodynamic potential, which comes from necessary regularizations inside the model, and only depends on the particles in the lowest Landau level.
- The second term $n_{ord} = 3|eB|\mu/2\pi^2$ is the corresponding ordinary non-anomalous contribution.
- Since $\mu > b$ in the region of interest, the leading contribution comes from the non-anomalous part,

$$n_q \simeq n_{ord} = \frac{3|eB|\mu}{2\pi^2} \tag{8}$$

[4] Ferrer, E.J. and de la Incera, V., *Nuclear Physics B*,. 931, pp. 192–215, 2018; Ferrer, E.J. and de la Incera, V., *Physics Letters B*,. 769, pp. 208–212, 2017.

The Heat Capacity of the MDCDW phase

- From the baryonic chemical potential,

$$\mu = \sqrt{k_F^2 + m_q^2} \tag{9}$$

Using the relation $\mu = k_B T_F$, where T_F is the Fermi temperature, and Eq.(8), we can rewrite the heat capacity

$$C_V^{MDCDW} \simeq \frac{\pi^2}{3} n_q k_B \left(\frac{T}{T_F} \right) \tag{10}$$

- where we include k_B^2, associated with the second derivative with respect to T in the definition of the heat capacity in Eq.(2).

The Heat Capacity of the MDCDW phase

- In order to estimate the heat capacity in this phase, we consider the baryonic number density $n_B = 3n_s$, where $n_s = 0.15 fm^{-3}$ is the saturation density, where hadrons are already touching each other.
- Given the baryonic number density it's possible to estimate the Fermi temperature, $T_F = 2.6 \times 10^{12} K$.
- Therefore, for $T = 10^8 K$

$$C_V^{MDCDW} = \frac{\pi^2}{3} n_q k_B \left(\frac{T}{T_F} \right) \simeq 0.26 \times 10^{19} \frac{erg}{Kcm^3} \qquad (11)$$

- Multiplying by the volume of a star with $R = 10km$, we find

$$\tilde{C}_V^{MDCDW} = C_V^{MDCDW} \times V_{NS} = 0.1 \times 10^{38} \frac{erg}{K} \qquad (12)$$

Conclusions

- We notice that the MDCDW phase heat capacity satisfies the lower-limit observational constraint $\tilde{C}_V^{MDCDW} \geqslant 10^{36}(\frac{T}{10^8})\frac{erg}{K}$.
- This result is a fundamental contribution to previous studies that pointed out that the MDCDW phase is plausible and stable at intermediate density[5] since in addition it predicts star-mass values in the range of those that have been observed[6].

[5] Ferrer, E.J. and de la Incera, V., *Nuclear Physics B*,. 931, pp. 192–215, 2018; Ferrer, E. J. and Incera, V. de la, *Phys. Rev. D*,. 102, p. 014010, 2020.

[6] Carignano, S. et al., *Phys. Rev. D*,. 92, p. 105018, 2015; Feng, Bo, Ferrer, Efrain J., and Portillo, Israel, *Phys. Rev. D*,. 101, p. 056012, 2020.

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