



Abstract

□ In this paper, we considered formation control of multiple WAM-Vs with uncertainty. It is assumed that there is parametric uncertainty and non-parametric uncertainty in the model of each WAM-V and the information of the leader WAM-V is available only to a portion of the follower WAM-Vs.

□ With the aid of backstepping techniques, Distributed robust tracking controllers are proposed.

□ To avoid calculation of the derivative of signals, Distributed command filtered controllers are also proposed.

□ Simulation results show the effectiveness of the proposed controllers.

Problem Statement

Kinematics of WAM-V

The kinematics of the j -th WAM-V can be written as

$$\dot{x}_j = u_j \cos \psi_j - v_j \sin \psi_j \quad (1)$$

$$\dot{y}_j = u_j \sin \psi_j + v_j \cos \psi_j \quad (2)$$

$$\dot{\psi}_j = r_j \quad (3)$$

Dynamics of WAM-V

The dynamics of the j -th WAM-V can be written as

$$m_{1j} \dot{u}_j - m_{2j} v_j r_j + d_{1j} u_j + D_{1j} = F_{Lj} + F_{Rj} \quad (4)$$

$$m_{2j} \dot{v}_j + m_{1j} u_j r_j + d_{2j} v_j + D_{2j} = 0 \quad (5)$$

$$m_{3j} \dot{r}_j - (m_{1j} - m_{2j}) u_j v_j + d_{3j} r_j + D_{3j} = L_j (F_{Lj} - F_{Rj}) \quad (6)$$

▪ m_{ij}, d_{ij} , and D_{ij} are unknown.

▪ The 0-th WAM-V is a leader and the other WAM-Vs are followers.

▪ The communication between WAM-V is described by a digraph

$$G^e = \{V^e, E^e\}$$

▪ It is given a desired geometric pattern P defined by constant vectors

$$[p_{jx}, p_{jy}]^T (1 \leq j \leq m) \text{ which satisfy } \sum_{j=1}^m p_{jx} = 0 \text{ and } \sum_{j=1}^m p_{jy} = 0$$

Formation Control Problem

Design a controller (F_{Lj}, F_{Rj}) for j -th follower WAM-V based on its neighbors' state information such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}, 1 \leq i, j \leq m \quad (7)$$

$$\lim_{t \rightarrow \infty} \sum_{j=1}^m \left(\frac{x_j}{m} - x_0 \right) = 0 \quad (8)$$

$$\lim_{t \rightarrow \infty} \sum_{j=1}^m \left(\frac{y_j}{m} - y_0 \right) = 0 \quad (9)$$

In the above problem,

eqn. (7) means that m follower WAM-Vs come into the desired geometric pattern P .

Eqn. (8)-(9) mean that the centroid of m follower WAM-Vs follows the leader WAM-V.

Proposed Controller (Solution)

A. Distributed Controller

With the aid of the backstepping technique, if the leader WAM-V is globally reachable distributed controllers are proposed as

$$F_{Rj} = \frac{1}{2} \left(\delta_{1j} + \frac{\delta_{2j}}{L_j} \right), \quad (10)$$

$$F_{Lj} = \frac{1}{2} \left(\delta_{1j} - \frac{\delta_{2j}}{L_j} \right) \quad (11)$$

Where,

$$\delta_{1j} = -K_3 z_{1j} + \bar{m}_{1j} \dot{\eta}_{3j} - \bar{m}_{2j} v_j r_j + \bar{d}_{1j} u_j - (\rho_{1j} |\dot{\eta}_{3j}| + \rho_{2j} |v_j r_j| - (\rho_{1j} |\dot{\eta}_{3j}| + \rho_{2j} |v_j r_j| + \rho_{4j} |u_j| + \rho_{6j}) \text{sign}(z_{1j})) \quad (12)$$

$$\delta_{2j} = -K_5 z_{3j} - z_{2j} - (\bar{m}_{1j} - \bar{m}_{2j}) u_j v_j + \bar{d}_{3j} r + \bar{m}_{3j} \dot{\eta}_{5j} - ((\rho_{1j} + \rho_{2j}) |u_j v_j| + \rho_{5j} |r_j| + \rho_{3j} |\dot{\eta}_{5j}| + \rho_{7j}) \text{sign}(z_{3j}) \quad (13)$$

B. Distributed Command Filtered Tracking Control

Laws

In the proposed controller in the last section, the derivatives of signals are needed. To avoid this, a command filtered controller can be designed

$$\dot{\chi}_{1j} = -r_{1j} |\chi_{1j} - \eta_{3j}|^{\frac{1}{2}} \text{sign}(\chi_{1j} - \eta_{3j}) + \chi_{2j} \quad (15)$$

$$\dot{\chi}_{2j} = -r_{2j} \text{sign}(\chi_{2j} - \dot{\chi}_{1j}) \quad (16)$$

$$\dot{\chi}_{3j} = -r_{3j} |\chi_{3j} - \eta_{4j}|^{\frac{1}{2}} \text{sign}(\chi_{3j} - \eta_{4j}) + \chi_{4j} \quad (17)$$

$$\dot{\chi}_{4j} = -r_{4j} \text{sign}(\chi_{4j} - \dot{\chi}_{3j}) \quad (18)$$

$$\dot{\chi}_{5j} = -r_{5j} |\chi_{5j} - \eta_{5j}|^{\frac{1}{2}} \text{sign}(\chi_{5j} - \eta_{5j}) + \chi_{6j} \quad (19)$$

$$\dot{\chi}_{6j} = -r_{6j} \text{sign}(\chi_{6j} - \dot{\chi}_{5j}) \quad (20)$$

Distributed command filtered controllers are

$$F_{Rj} = \frac{1}{2} \left(\delta_{1j} + \frac{\delta_{2j}}{L_j} \right), \quad (23)$$

$$F_{Lj} = \frac{1}{2} \left(\delta_{1j} - \frac{\delta_{2j}}{L_j} \right) \quad (24)$$

Where

$$\delta_{1j} = -K_3 (u_j - \chi_{1j}) + \bar{m}_{1j} \dot{\chi}_{1j} - \bar{m}_{2j} v_j r_j + \bar{d}_{1j} u_j - (\rho_{1j} |\dot{\chi}_{1j}| + \rho_{2j} |v_j r_j| + \rho_{4j} |u_j| + \rho_{6j}) \text{sign}(w_{1j}) \quad (25)$$

$$\delta_{2j} = -K_5 (r_j - \chi_{5j}) - w_{2j} - (\bar{m}_{1j} - \bar{m}_{2j}) u_j v_j + \bar{d}_{3j} r + \bar{m}_{3j} \dot{\chi}_{5j} - ((\rho_{1j} + \rho_{2j}) |u_j v_j| + \rho_{5j} |r_j| + \rho_{3j} |\dot{\chi}_{5j}| + \rho_{7j}) \text{sign}(z_{3j}) \quad (26)$$

Graphical Representation

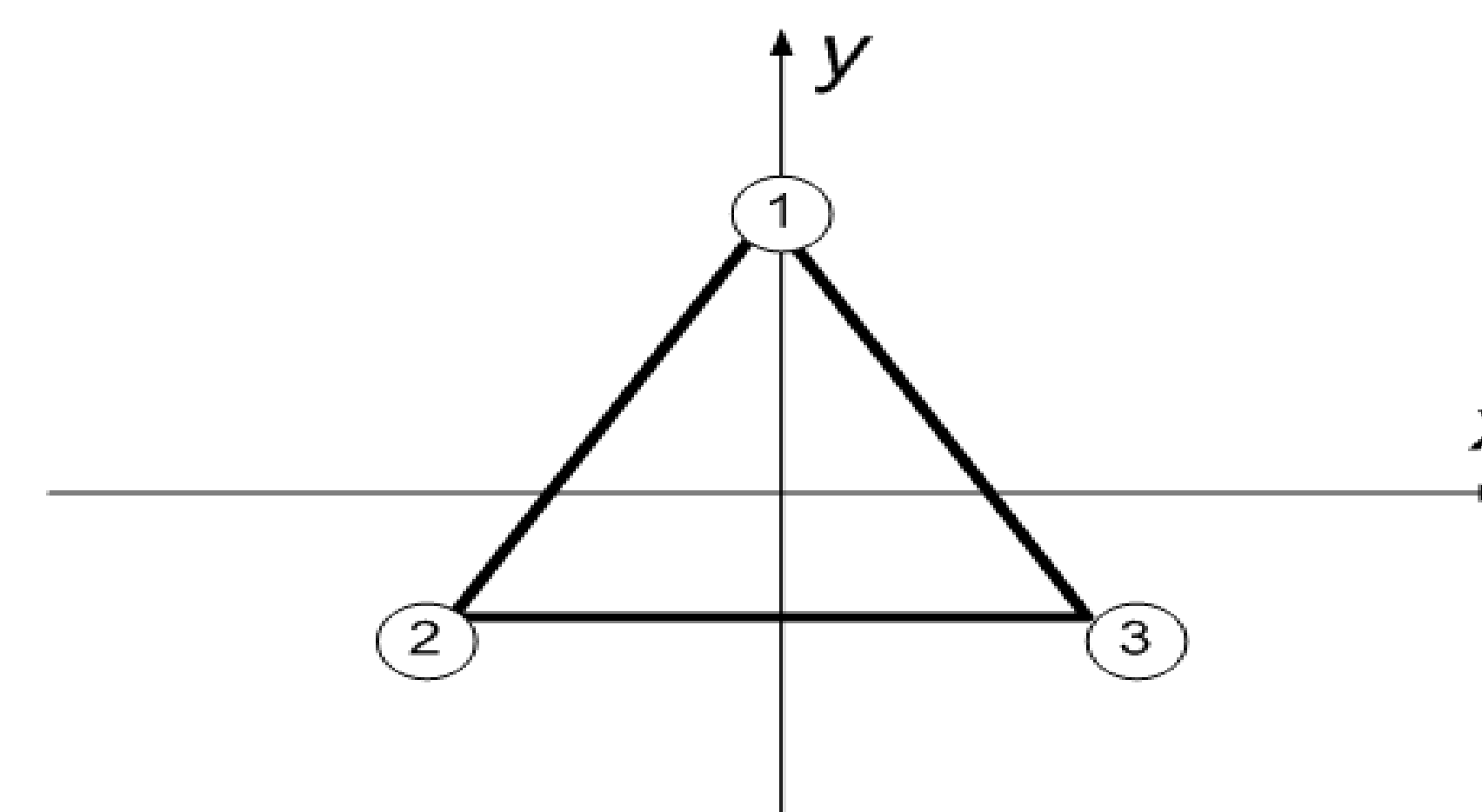


Fig 1: Desired formation of three WAM-Vs

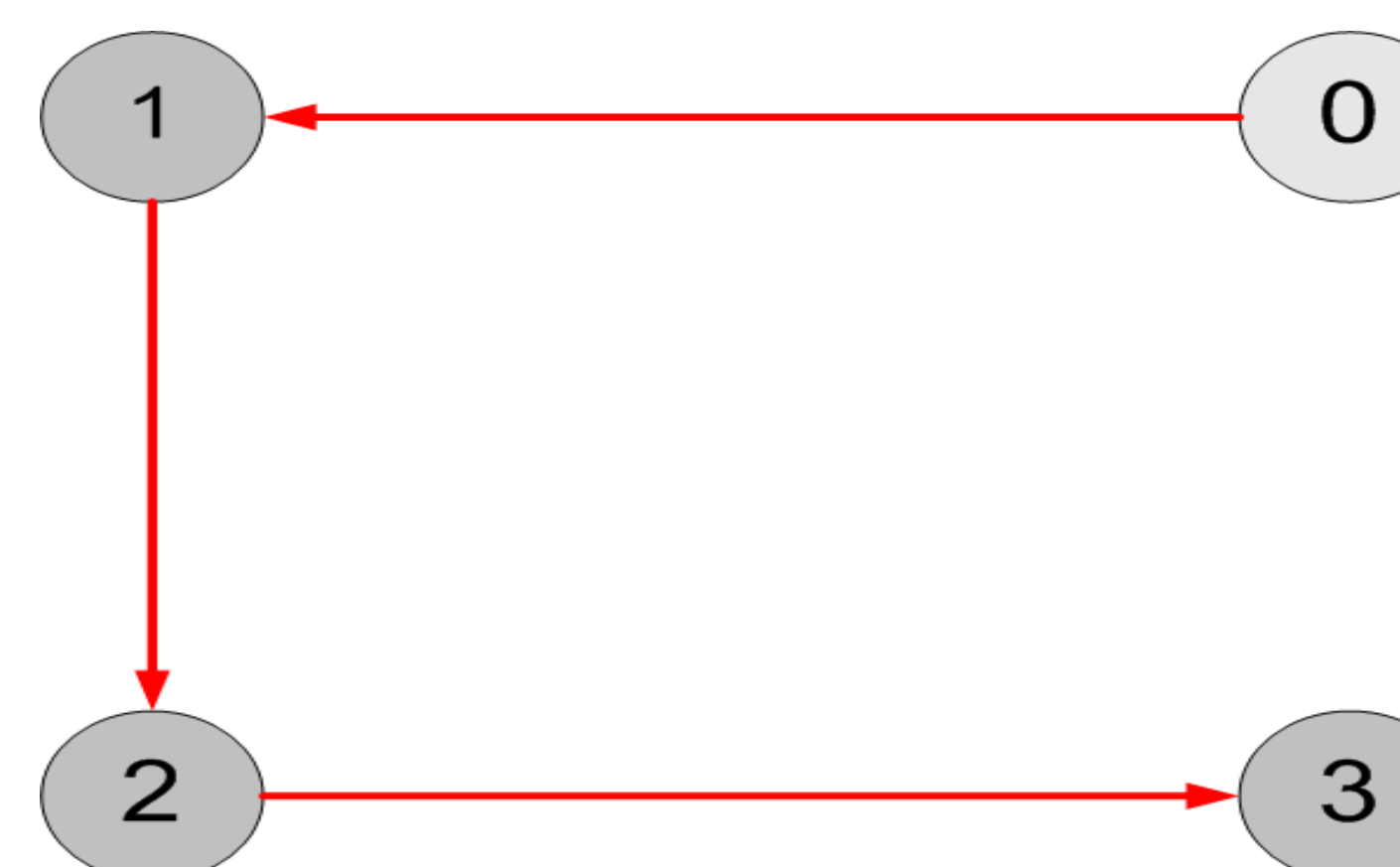


Fig 2: Communication digraph

Simulation Result

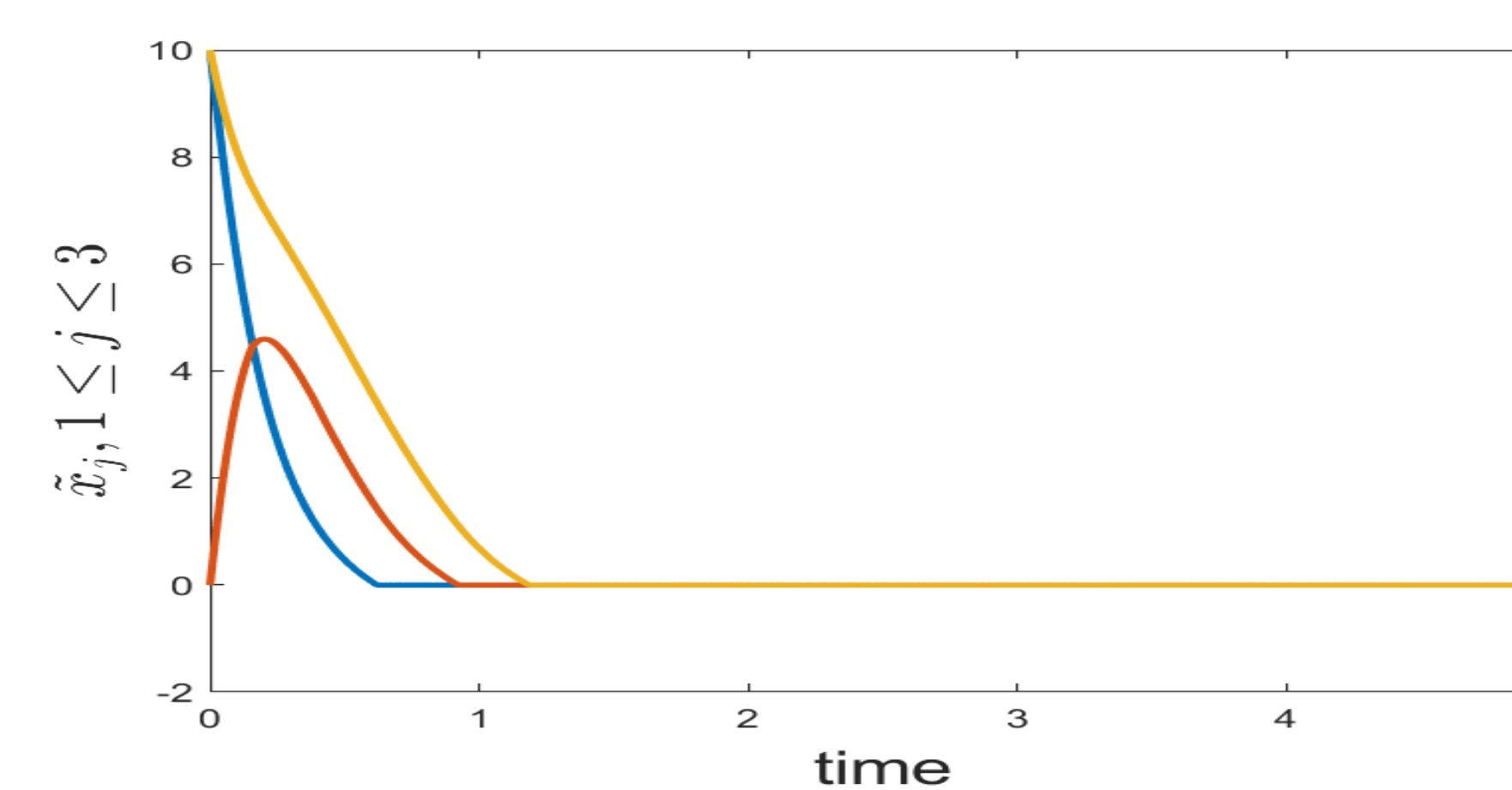


Fig. 3: Time response of \tilde{x}_j for $1 \leq j \leq 3$

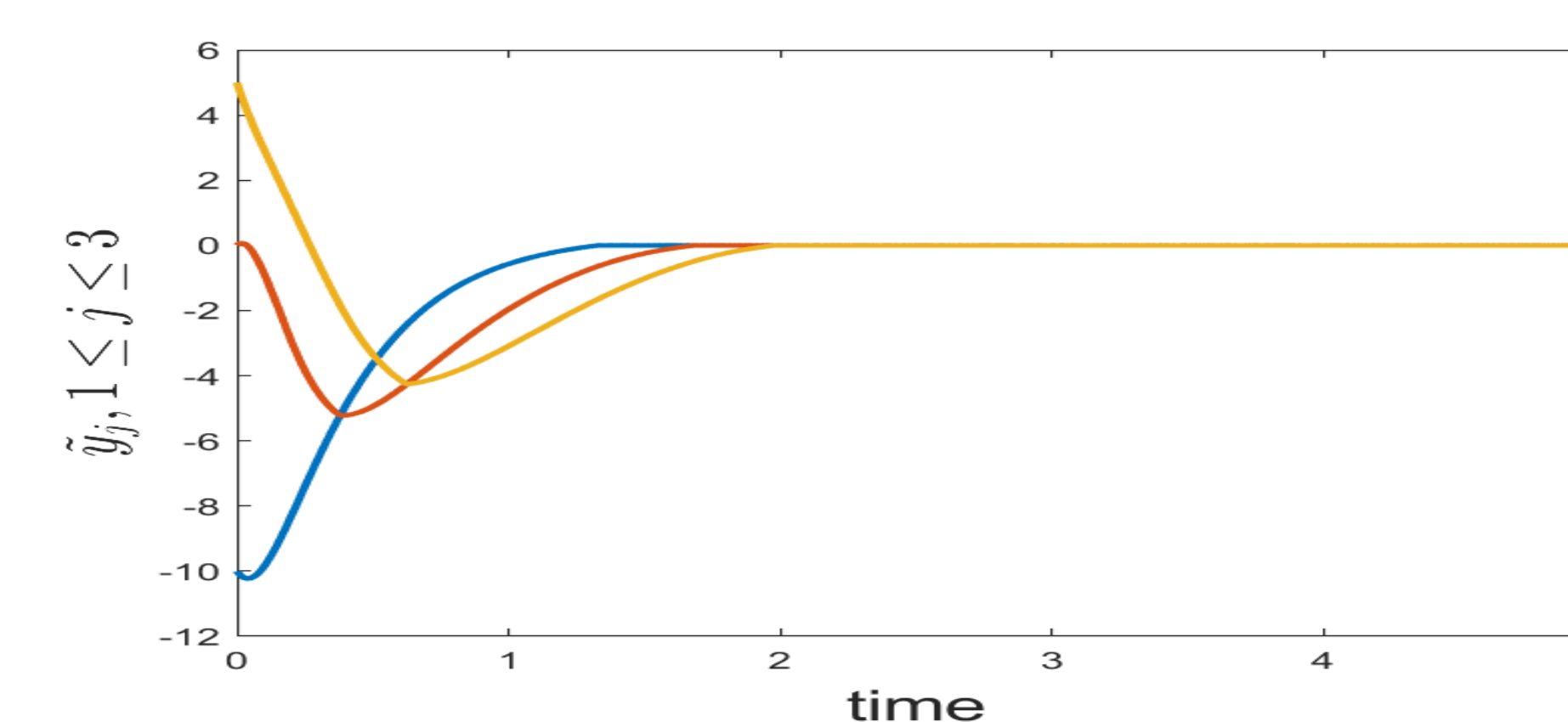


Fig. 4: Time response of \tilde{y}_j for $1 \leq j \leq 3$

Simulation Result (cont.)

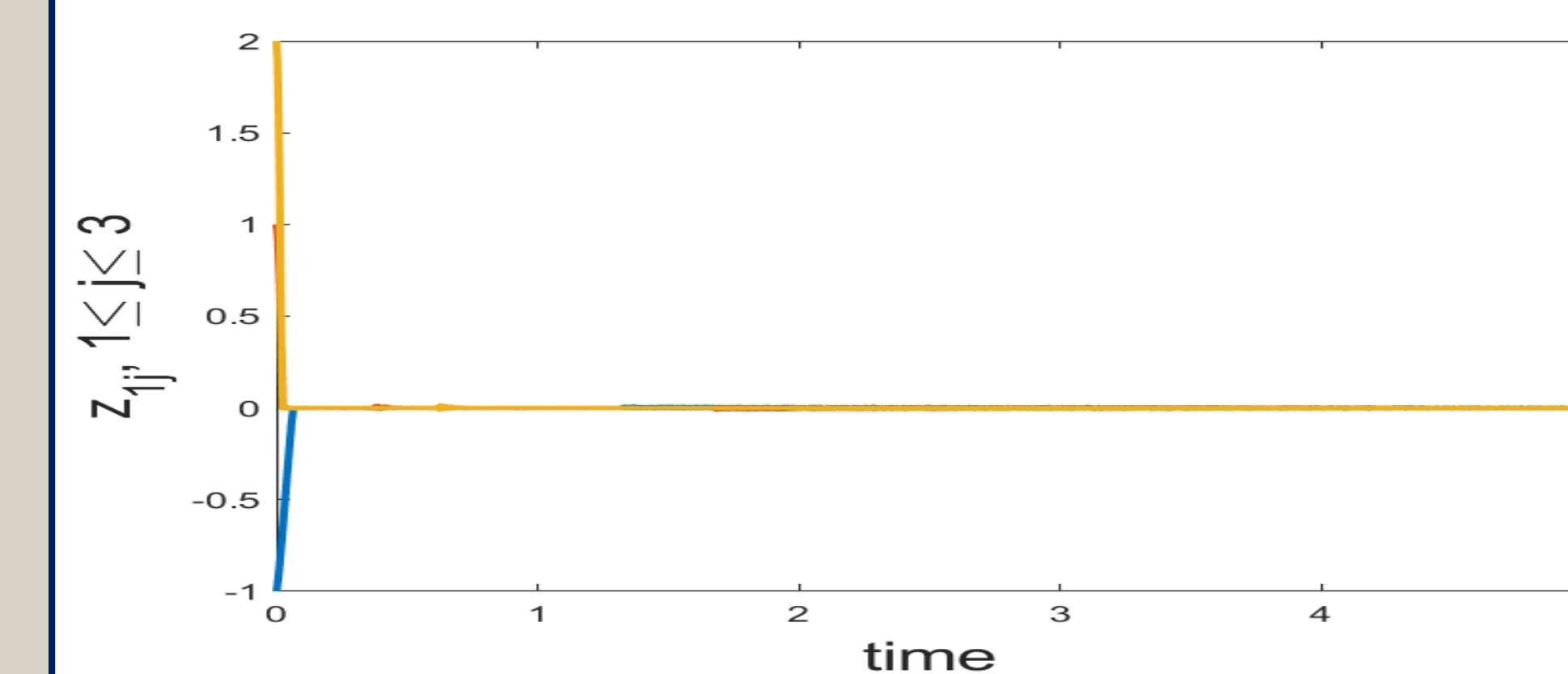


Fig. 5: Time response of z_{1j} for $1 \leq j \leq 3$

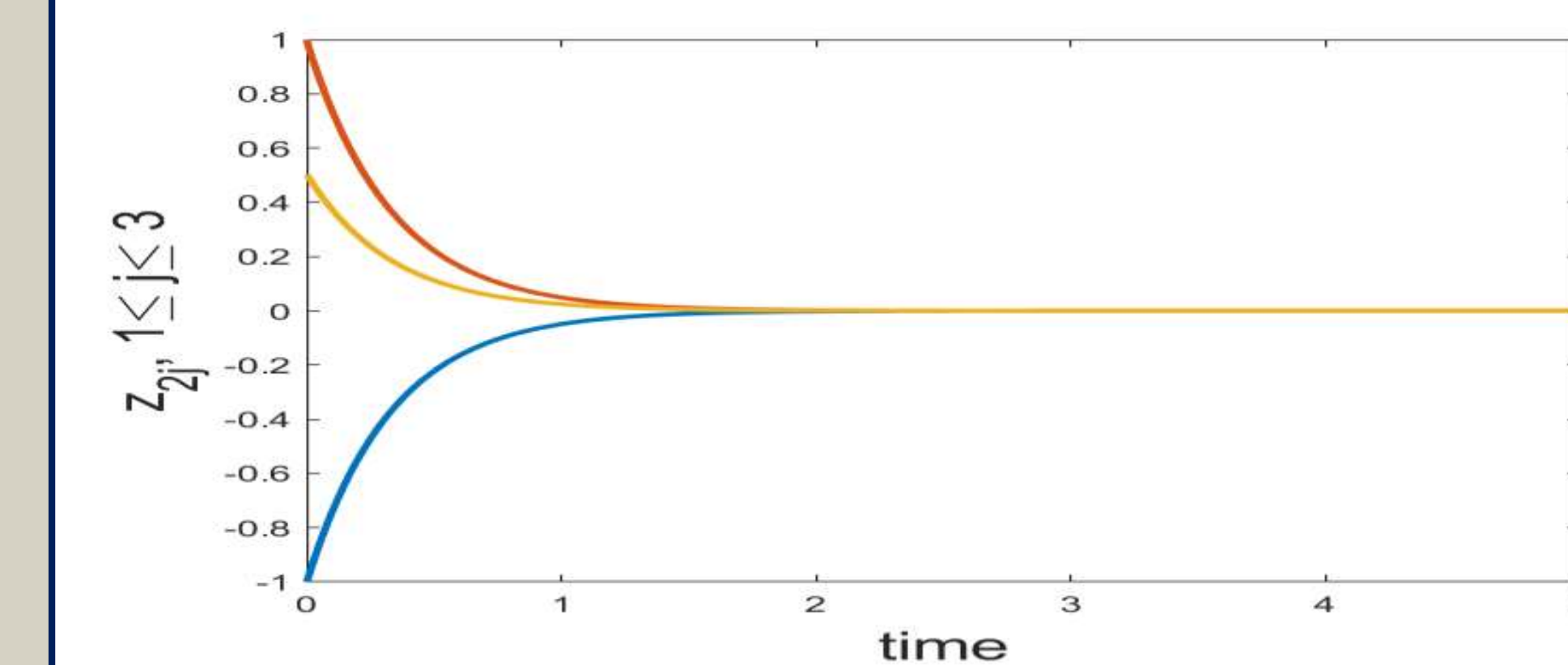


Fig 6: Time response of z_{2j} for $1 \leq j \leq 3$

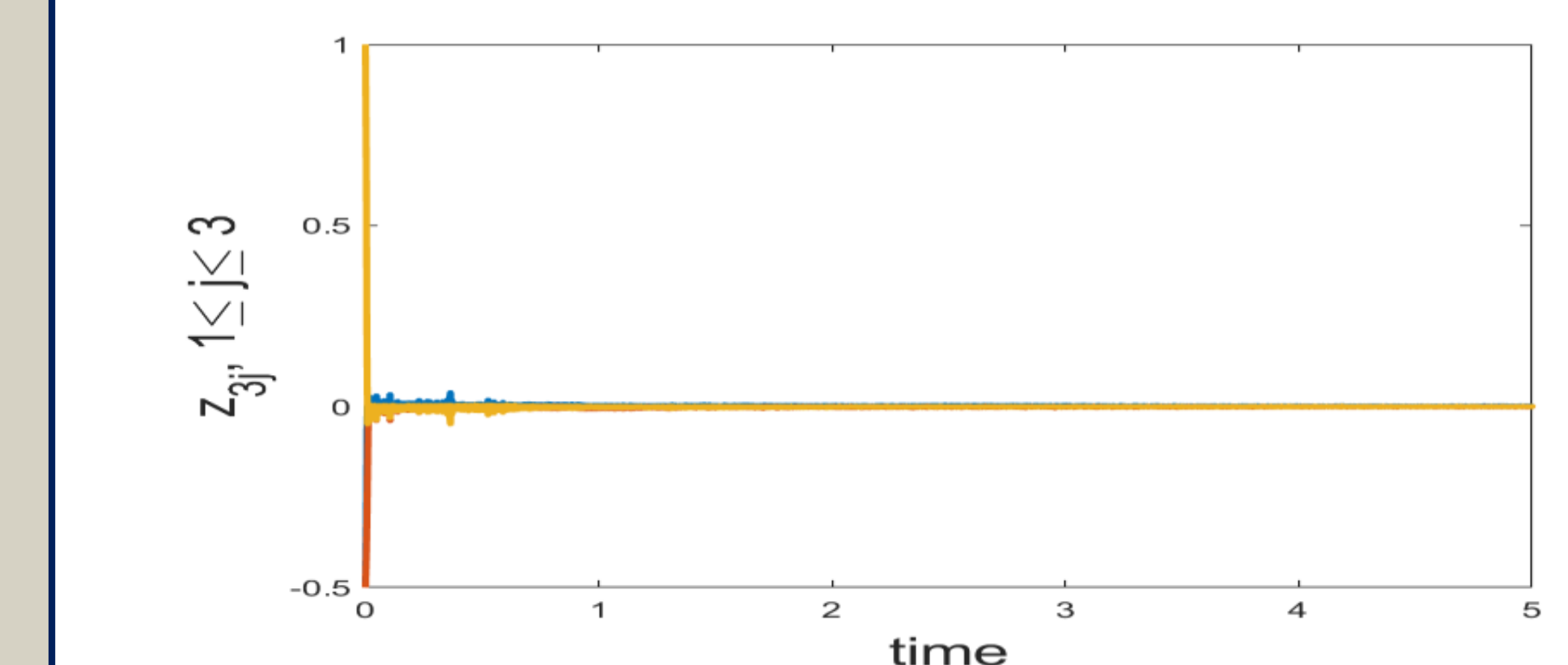


Fig.7: Time response of z_{3j} for $1 \leq j \leq 3$

Conclusion

- ✓ In this paper, we considered formation control of multiple uncertain WAM-Vs with a leader WAM-V.
- ✓ If inertia parameters are not exactly known, distributed robust tracking laws were proposed with the aid of neighbors' information.
- ✓ To reduce the computation load in controller design, distributed command filtered controllers were proposed. Simulation results show the effectiveness of the proposed results.

References

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