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Distributed cooperative control of multiple UAVs with uncertainty

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Abstract

This paper considers the formation flying of multiple quadrotors with a desired orientation and a leader. In the formation flying control, it is assumed that the desired formation is time-varying and there are the system uncertainty and the information uncertainty. In order to deal with different uncertainties, a backstepping-based approach is proposed for the controller design. In the proposed approach, different types of uncertainties are considered in different steps. By integrating adaptive/robust control results and Laplacian algebraic theory, distributed robust adaptive control laws are proposed such that the formation errors exponentially converge to zero and the attitude of each quadrotor exponentially converges to the desired value. Simulation results show the effectiveness of the proposed algorithms.

Keywords Quadrotor · Distributed control · Cooperative control · Leader–follower control · Formation control

1 Introduction

Formation flying of multiple quadrotors has been studied recently due to its wide applications in civil and military applications, such as surveillance, area exploration, target search, accident rescue tasks, and many other applications. The capacity of vertical taking-off and landing makes quadrotors superior to other unmanned aerial vehicles. Compared with a single quadrotor, the formation of multiple quadrotors can perform more difficult tasks and provides better performance. However, the underactuated nature of a single quadrotor makes the cooperative control of multiple quadrotors challenging.

Formation control of multiple quadrotors is to coordinate a group of quadrotors to achieve a desired spatial geometric pattern. In the past decades, several classical approaches have been proposed for multi-agent systems, which include the behavioral approach, the virtual structure approach, the leader-follower approach, and the graph theoretical approach. In the leader-following approach [1,2], some agents are designated as leaders and the others are designated as followers. The leaders track the predefined trajectories and the followers track the state of their neighbors according to given schemes. In the behavioral approach [3–5], the control action for each agent is defined by a weighted average of the control corresponding to each desired behavior for the agent. In the virtual structure approach [6–8], the entire formation is treated as a single rigid body. The virtual structure moves along a desired trajectory and with a desired attitude. In the graph theoretical approach [9–12], each agent is considered as a node and the communication between agents is defined by a graph. The control law is designed with the aid of the difference of neighbors' information.

Formation control of multiple unmanned aerial vehicles (UAVs) has been studied extensively. In [13,14], the dynamics of each vehicle is simplified as a linear system and the formation control is studied based on multiple linear systems. In [15–17], the formation control was studied based on the translational and rotational motions with linearized or simplified models. Noting that a UAV is a multiple-input multiple-output system with highly nonlinear and strongly coupled dynamics and has 6 degrees of freedom (6-DOF), formation control of multiple UAV was studied based on 6-DOF dynamics in [18–21]. In [19,20], formation control of multiple UAVs was studied based on the 6-DOF model with disturbances and robust distributed control laws were proposed. In [21], distributed formation control of multiple UAVs was studied based on a nonsmooth backstepping design and consensus techniques for the 6-DOF models.

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The formation controllers in the literature mentioned above ensure that the states of a UAV asymptotically converge to a desired formation as time goes to infinity. In practical applications, finite-time distributed controllers are preferred because they ensure that the states of a UAV converge to a desired formation within a finite time and the closed-loop systems have better disturbance rejection performance. In [22], formation control of multiple UAVs with nonparametric uncertainties was studied. Finite-time controllers were proposed with the aid of finite-time distributed observers. In [16], finite-time distributed controllers were proposed based on the linearized models without uncertainty with the aid of the properties of homogeneous systems. In [16,22], the attitudes of UAVs are defined by Euler angles. To make the attitude control laws nonsingular, the Euler angles are limited to some intervals.

Although there are many results on formation control of multiple UAVs, how to improve the control performance is still challenging in the presence of uncertainty and coupling among neighboring UAVs. Motivating by the research work mentioned above and the work in [23–25], in this paper we study the formation control of multiple UAVs with parametric and nonparametric uncertainties and propose new distributed control laws such that the formation errors exponentially converge to zero and the attitude of each UAV exponentially converges to a desired attitude. In order to solve the formation control problem, a multi-step *backstepping-based approach* is proposed and distributed exponential control laws are designed. The proposed approach includes six steps. In the first step an auxiliary system for each quadrotor is introduced to estimate the parametric and nonparametric uncertainties in the dynamics of each quadrotor. In the second step, distributed kinematic controllers are proposed for the translation with the aid of the graph theory and the formation problem is solved without the information whether the leader's information is available to a quadrotor or not. In the third step, distributed dynamic controllers for the translation and the desired attitude for each quadrotor are designed based on the backstepping technique. In the fourth step, the force input and the desired attitude for each quadrotor are calculated. In the fifth step, a distributed kinematic controller for the attitude control of each quadrotor is proposed with the aid of the graph theory. In the last step, a dynamic controller for the attitude of each quadrotor is proposed with the aid of the backstepping technique. In the proposed approach, the uncertainties in the dynamics and the uncertainty of the leader's information are considered separately in different steps. With the aid of this multi-step approach, distributed robust adaptive controllers are proposed such that multiple quadrotors exponentially converge to a desired formation and the Y-axis of each quadrotor exponentially converges to the desired direction. Compared to the results in literature, the contributions of this paper are as follows.

- This paper solves the formation control problem of multiple quadrotors in a more general setting. In the considered problem, the formation is time-varying and there are both parametric uncertainty and nonparametric uncertainty in the dynamics of each quadrotor. Furthermore, the communications among quadrotors are directional, which means that the information exchange between two quadrotors is one-way instead of two-way.
- A new systematic multi-step controller design approach is proposed for the formation control problem by integrating the uncertainty decomposition technique and the backstepping technique. In this approach, different types of uncertainties are dealt with in different steps. The difficulty of the controller design is greatly reduced.
- The proposed control laws ensure that a group of quadrotors exponentially converge to a desired formation with a desired orientation, which means that the proposed control systems have better performance in convergence and disturbance rejection. Moreover, the proposed controllers are distributed. No global information is required in the controllers.

The remaining part of this paper is organized as follows. In Sect. 2, the considered problem is defined and some preliminary results are presented. In Sect. 3, a multi-step approach is proposed and distributed controllers are derived. In Sect. 4, simulation results are presented. The last section concludes this paper.

2 Problem statement and preliminaries

2.1 Problem statement

Consider m quadrotors. Under some assumptions, the kinematics and dynamics of j-th quadrotor are defined by

$$\dot{p}_j = v_j \tag{1}$$

$$\dot{v}_j = -ge_3 + \frac{1}{m_j} f_j R_j e_3 + d_{1j} \tag{2}$$

$$\dot{R}_j = RS(\omega_j) \tag{3}$$

$$J_j \dot{\omega}_j = S(J_j\omega_j)\omega_j + \tau_j + d_{2j} \tag{4}$$

where p_j and v_j are the position and the velocity of the mass center in the inertia frame, respectively, g is the gravitational acceleration, $e_3 = [0, 0, 1]^\top$, $f_j \in \Re$ is the total thrust, $R_j = [b_{1j}, b_{2j}, b_{3j}]$ is the rotation matrix of the body frame with respect to the inertia frame, ω_j is the angular velocity of the quadrotor in its body frame, J_j is the inertia moment of the quadrotor, d_{1j} and d_{2j} denote nonparametric uncertainty and disturbance, $S(\xi)$ for $\xi = [\xi_1, \xi_2, \xi_3]^\top$ is a skew-symmetric

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matrix defined by

$$S(\xi)=\begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix},$$

and $\tau_j = [\tau_{1j}, \tau_{2j}, \tau_{3j}]^\top$ is the torque input of the system.

For multiple quadrotors, there are information flows between them with the aid of sensors or wireless communication. Consider each quadrotor as a node. The communication between quadrotors is defined by a directed graph $\mathcal{G} = \{\mathcal{A}, \mathcal{E}\}$ where $\mathcal{A}$ is the node set and $\mathcal{E}$ is the edge set. If there is an edge e_{ij} in $\mathcal{E}$ it means that the information of node i is available to node j. Node i is called a neighbor of node j if the information of node i is available to node j. All neighbors of node j form a node set which is called the neighbor set of node j and is denoted by $\mathcal{N}_j$. A directed path from node i to node j is a sequence of sets of edges that connect node i to node j by following their directions. Node i is said to be reachable to node j if there exists a directed path from node i to node j. Node i is said globally reachable if node i is reachable for every other node in $\mathcal{A}$.

In this paper, we assume there are m follower quadrotors and one leader quadrotor. The leader quadrotor is operated by a human operator and does not receive any information from the follower quadrotors. Without loss of generality, the leader quadrotor is labeled as node 0. The follower quadrotors are labeled by $1, 2, \ldots, m$. The communication between $m+1$ quadrotors is defined by an augmented directed graph $\mathcal{G}^a = \{\mathcal{A}^a, \mathcal{E}^a\}$ where $\mathcal{A}^a = \mathcal{A} \cup \{0\}$ and $\mathcal{E}^a$ is a union of $\mathcal{E}$ and the edges from node 0 to the followers.

For m follower quadrotors and a leader quadrotor, a desired formation can be defined by $(m+1)$ vectors $h_j \in R^3$ which may be constant vectors or time-varying vectors. We say $(m+1)$ quadrotors are in the desired formation if

$$p_i - p_j = h_i - h_j$$

for any $0 \le i, j \le m$. We say $m+1$ quadrotors come into the desired formation if

$$\lim_{t\to\infty} [(p_i - h_i) - (p_j - h_j)] = 0$$

for any $0 \le i, j \le m$.

In the dynamics (1–4), the parametric uncertainty (i.e., m_j and J_j) and nonparametric uncertainty (i.e., d_{1j} and d_{2j}) are called the *system uncertainty*. For each quadrotor, it is unknown whether the leader quadrotor is a neighbor or not. We say there is *information uncertainty* for each quadrotor.

In this paper, we consider the following control problem.

2.1.1 Formation flying with a leader

For a leader quadrotor and m follower quadrotors, it is assumed that m_j, J_j, d_{1j}, and d_{2j} are unknown for $1 \le j \le m$. It is given a desired formation defined by h_j for $0 \le j \le m$, the control problem is to design distributed state feedback controllers f_j and τ_j using its own information and its neighbors' information such that

$$\lim_{t\to\infty} [(p_j(t) - h_j(t)) - (p_i(t) - h_i(t))] \overset{exp.}{=} 0 \tag{5}$$

$$\lim_{t\to\infty} (b_{2j}(t) - b_{2,0}(t)) \overset{exp.}{=} 0 \tag{6}$$

where $\overset{exp.}{=}$ means "exponentially converges to".

In the defined problem, (5) means that the $(m+1)$ quadrotors come into the desired formation and (6) means that the Y-axes of the body frames of $m+1$ quadrotors are parallel as time goes to infinity.

In order to solve the defined problem, the following assumptions are made.

Assumption 1 The mass m_j of quadrotor j is an unknown constant and $\underline{m}_j \le m_j \le \bar{m}_j$ where $\underline{m}_j$ and $\bar{m}_j$ are known constants.

Assumption 2 d_{1j} and d_{2j} are continuous functions of the system state and the time and are bounded.

Assumption 3 The communication graph $\mathcal{G}^a$ is a directed graph and the node 0 is globally reachable.

Assumption 4 $b_{2,0}(t)$ is smooth. $\dot{b}_{2,0}$ and $\ddot{b}_{2,0}$ are bounded. $b_{2,0}^\top(t) b_{3,0}(t) = 0$ for any time where $b_{3,0}(t) = \frac{\ddot{p}_0(t)+ge_3}{\|\ddot{p}_0(t)+ge_3\|_2}$.

Assumption 1 is reasonable in practice because the mass of a quadrotor is always bounded by some constant. Since d_1 and d_2 are friction and disturbance, it is reasonable to assume that they are bounded in Assumption 2.

Assumption 4 is due to the motion of the quadrotor in (1–2). In Assumption 4, $b_{3,0}$ is obtained as follows. If the quadrotor moves along the desired trajectory, by (1–2) one has

$$R_0 e_3 = \frac{f(\ddot{p}_0 + ge_3)}{m} = \frac{\ddot{p}_0 + ge_3}{\|\ddot{p}_0 + ge_3\|_2}.$$

Since m_j and f_j are not zero and $R_0 e_3$ is an unit vector, $b_{3,0}$ should be the third column of R_0.

2.2 Kinematics of rotation using quaternion

The attitude of j-th quadrotor can be defined by an unit quaternion $q_j = \left[\eta_j, \epsilon_j^\top\right]^\top$ where $\eta_j \in \Re$ and $\epsilon_j \in \Re^3$. The relation between q_j and the rotation matrix R_j is defined by

$$R_j = \mathcal{R}(q_j) = I + 2\eta_j S(\epsilon_j) + 2S^2(\epsilon_j).$$

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Noting that for any rotation matrix R, there are exactly two unit quaternions, $\pm q$, such that $R = \mathcal{R}(q) = \mathcal{R}(-q)$.

For j-th quadrotor, (3) can be written as

$$\dot{q}_j = \frac{1}{2}A(q_j)\omega_j \tag{7}$$

where

$$A(q_j) = \begin{bmatrix} -\epsilon_j^\top \\ \eta_j I + S(\epsilon_j) \end{bmatrix}. \tag{8}$$

2.3 Notations and preliminary results

Let $\mathcal{L}_\infty$ denote bounded functions and $\mathcal{L}_2$ denote square integrable functions.

For $x \in \Re$, we define the function

$$\chi(x) = \frac{x}{\sqrt{x^2 + e^{-2\kappa t}}} \tag{9}$$

where $\kappa > 0$. If $x = [x_1, \ldots, x_l]^\top \in \Re^l$, $\chi(x)$ is defined as

$$\chi(x) = [\chi(x_1), \ldots, \chi(x_l)]^\top.$$

It can be proved that

$$x^\top \chi(x) \geq \sum_{i=1}^{l} |x_i| - le^{-\kappa t}.$$

The results in the following lemma are useful.

Lemma 1 *Consider $m+1$ agents, where agent 0 is the leader agent and agent j is a follower agent for $1 \leq j \leq m$. The state of the agent j is x_j. The communication among agents is defined by a direct graph $\mathcal{G}^a$ and it is assumed that the state x_0 is globally reachable to all other agents.*

(1) If $\dot{x}_0 = 0$ and the state of agent j for $1 \leq j \leq m$ is defined by

$$\dot{x}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) + u_j$$

then the system with input u and state $\tilde{x}$ has the input-to-state stability (ISS) property [26], where $u = [u_1^\top, \ldots, u_m^\top]^\top$ and $\tilde{x} = [x_1^\top - x_0^\top, \ldots, x_m^\top - x_0^\top]^\top$. Moreover, if u exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.

(2) If $\max_{t\in[0,\infty)} |\dot{x}_0(t)| \leq \nu$ and the state of agent j for $1 \leq j \leq m$ is defined by

$$\dot{x}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) - \nu\chi\left(\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i)\right) + u_j$$

then the system with input u and state $\tilde{x}$ has the ISS property, where u and $\tilde{x}$ are defined in item 1. Moreover, if u exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.

(3) If $\max_{t\in[0,\infty)} |\dot{x}_0(t)| \leq \nu_0$ and the state of agent j for $1 \leq j \leq m$ is defined by

$$\begin{aligned} \dot{x}_j &= -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) - \nu_j\chi\left(\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i)\right) \\ &\quad + u_j \\ \dot{\nu}_j &= -\sum_{i \in \mathcal{N}_j^a} a_{ji}(\nu_j - \nu_i) \end{aligned}$$

then $\lim_{t\to\infty}(\nu_j - \nu_0) \overset{exp.}{=} 0$ and the system with input u and state $\tilde{x}$ has the ISS property, where u and $\tilde{x}$ are defined in item 1. Moreover, if u exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.

The lemma can be proved based on the results of properties of Laplacian matrices and ISS properties and is omitted here.

3 Controller design

The presence of the system uncertainty and the information uncertainty makes the distributed controller design extremely hard when the communication graph is directed. In order to solve the defined problem, we propose the following multi-step controller design approach in which different types of uncertainty are dealt with in different steps.

Step 1: In the dynamics of the translation (1–2), m_j is an unknown constant and d_{1j} is an unknown time-varying function. We use a two-layer neural network to learn d_{1j}. Based on the universal approximation property of neural networks [27], there is a basis matrix ϕ_{1j} and an optimal weighted vector θ_{1j} with appropriate dimensions such that

$$d_{1j} = \phi_{1j}\theta_{1j} + \epsilon_{1j} \tag{10}$$

where ϵ_{1j} is the approximation error vector and $\|\epsilon_{1j}\| \leq \delta_{1j}$.

In order to deal with the system uncertainty, an auxiliary system for j-th quadrotor ($1 \leq j \leq m$) is introduced as follows.

$$\dot{z}_{1j} = z_{2j} + L_{1j}(p_j - z_{1j}) \tag{11}$$

$$\begin{aligned} \dot{z}_{2j} &= -ge_3 + \beta_j f_j R_j e_3 + \phi_{1j}\hat{\theta}_{1j} + L_{2j}(v_j - z_{2j}) \\ &\quad + (p_j - z_{1j}) + \delta_{1j}\chi(e^{\lambda t}(v_j - z_{2j})) \end{aligned} \tag{12}$$

where λ, L_{1j} ($> \lambda$) and L_{2j} ($> \lambda$) are positive constants, $\kappa > \lambda$, β_j is an estimate of $\frac{1}{m_j}$, and $\hat{\theta}_{1j}$ is an estimate of

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θ_{1j} and will be designed later. Let $e_{1j} = e^{\lambda t}(p_j - z_{1j})$ and $e_{2j} = e^{\lambda t}(v_j - z_{2j})$, then

$$\dot{e}_{1j} = e_{2j} - (L_{1j} - \lambda)e_{1j}$$
$$\dot{e}_{2j} = \left(\frac{1}{m_j} - \beta_j\right) e^{\lambda t} f_j R_j e_3 + e^{\lambda t}\phi_{1j}(\theta_{1j} - \hat{\theta}_{1j}) + e^{\lambda t}\epsilon_{1j} - e_{1j} - (L_{2j} - \lambda)e_{2j} - \delta_{1j}e^{\lambda t}\chi(e_{2j})$$

To make e_{1j} converge to zero, we choose a Lyapunov function candidate

$$V_{1j} = \frac{1}{2}e_{1j}^{\top}e_{1j} + \frac{1}{2}e_{2j}^{\top}e_{2j} + \frac{\gamma_{1j}^{-1}}{2}\left(\frac{1}{m_j} - \beta_j\right)^2 + \frac{\gamma_{2j}^{-1}}{2}(\theta_{1j} - \hat{\theta}_{1j})^{\top}(\theta_{1j} - \hat{\theta}_{1j})$$

where γ_{1j} and γ_{2j} are positive constants. The derivative of V_{1j} is

$$\begin{aligned}\dot{V}_{1j} = &-e_{1j}^{\top}(L_{1j} - \lambda)e_{1j} + e_{2j}^{\top}\left(\frac{1}{m_j} - \beta_j\right) e^{\lambda t} f_j R_j e_3 \\ &+ e_{2j}^{\top}e^{\lambda t}\phi_{1j}(\theta_{1j} - \hat{\theta}_{1j}) + e_{2j}^{\top}e^{\lambda t}\epsilon_{1j} \\ &-e_{2j}^{\top}(L_{2j} - \lambda)e_{2j} - \gamma_{1j}^{-1}\left(\frac{1}{m_j} - \beta_j\right)\dot{\beta}_j \\ &- \gamma_{2j}^{-1}(\theta_{1j} - \hat{\theta}_{1j})^{\top}\dot{\hat{\theta}}_j - e_{2j}^{\top}\delta_{1j}e^{\lambda t}\chi(e_{2j})\end{aligned}$$

We choose

$$\dot{\beta}_j = Proj_{\Omega_j}[\gamma_{1j}e_{2j}^{\top}e^{\lambda t} f_j R_j e_3] \tag{13}$$
$$\dot{\hat{\theta}}_{1j} = \gamma_{2j}e^{\lambda t}\phi_{1j}^{\top}e_{2j} \tag{14}$$

where $Proj_{\Omega_j}$ denotes the projection to $\Omega_j = \left[\frac{1}{\overline{m}_j}, \frac{1}{\underline{m}_j}\right]$ [28], then

$$\begin{aligned}\dot{V}_{1j} = &-e_{1j}^{\top}(L_{1j} - \lambda)e_{1j} + e_{2j}^{\top}e^{\lambda t}\epsilon_{1j} - e_{2j}^{\top}(L_{2j} - \lambda)e_{2j} \\ &- \delta_j e_{2j}^{\top}e^{\lambda t}\chi(e_{2j}) \\ \le &-e_{1j}^{\top}L_{1j}e_{1j} - e_{2j}^{\top}L_{2j}e_{2j} + 3\delta_{1j}e^{-(\kappa-\lambda)t}.\end{aligned} \tag{15}$$

Lemma 2 *For the systems in (1–2) and the auxiliary system in (11–12) with update laws in (13–14), the estimates β_j and $\hat{\theta}_{1j}$ are bounded and*

$$\lim_{t\to\infty}(p_j - z_{1j}) \overset{exp.}{=} 0 \tag{16}$$
$$\lim_{t\to\infty}(v_j - z_{2j}) \overset{exp.}{=} 0. \tag{17}$$

Proof For the Lyapunov function V_{1j}, we have (15). Integrating both sides of (15), we have

$$V_{1j}(t) \le V_{1j}(0) + \frac{3\delta_j}{\kappa - \lambda} - \frac{3\delta_{1j}}{\kappa - \lambda}e^{-(\kappa-\lambda)t} < \infty$$

So, $V_{1j} \in \mathcal{L}_\infty$. By the definition of V_{1j}, $\beta_j, \hat{\theta}_j, e_{1j}$ and e_{2j} are bounded. Noting the definitions of e_{1j} and e_{2j}, z_{1j} and z_{2j} exponentially converge to zero. So, (16–17) are satisfied. □

For the leader quadrotor, an auxiliary system is not required. For convenience, we define

$$z_{10} = p_0, \quad z_{20} = v_0.$$

Step 2: Noting that $p_{1j} - z_{1j}$ exponentially converges to zero, (5) is satisfied if

$$\lim_{t\to\infty}[(z_{1j} - h_j) - (z_{1i} - h_i)] \overset{exp.}{=} 0, \quad 0 \le i, j \le m. \tag{18}$$

We assume that z_{2j} is a virtual control input and design it for the system in (11–12) such that (18) is satisfied.

Let

$$\tilde{z}_{1j} = z_{1j} - h_j - (z_{10} - h_0),$$
$$\tilde{z}_{2j} = z_{2j} - \dot{h}_j - (z_{20} - \dot{h}_0)$$

for $0 \le j \le m$, then

$$\dot{\tilde{z}}_{1j} = \tilde{z}_{2j} + L_{1j}(p_j - z_{1j}), \quad 1 \le j \le m \tag{19}$$

If $\tilde{z}_{2j}$ is a virtual input, the system in (19) can be considered as a linear system with an additional term. We choose the virtual control for $\tilde{z}_{2j}$ as

$$\begin{aligned}\alpha_{1j} &= -\sum_{i\in\mathcal{N}_j^a} a_{ji}(\tilde{z}_{1j} - \tilde{z}_{1i}) - L_{1j}(p_j - z_{1j}) \\ &= -\sum_{i\in\mathcal{N}_j^a} a_{ji}(z_{1j} - h_j - z_{1i} + h_i) - L_{1j}(p_j - z_{1j}), \quad 1 \le j \le m\end{aligned} \tag{20}$$

where $a_{ji} > 0$. With the aid of the virtual control α_{1j}, we have

$$\dot{\tilde{z}}_{1j} = -\sum_{i\in\mathcal{N}_j^a} a_{ji}(\tilde{z}_{1j} - \tilde{z}_{1i}) + \tilde{z}_{2j} - \alpha_{1j}. \tag{21}$$

For the systems in (21), the following results can be proved with the aid of Lemma 1 and its proof is omitted.

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Lemma 3 *For the systems in (21), under Assumption 1, if* $\tilde{z}_{2j} - \alpha_{1j}$ *exponentially converges to zero for* $1 \le j \le m$, *then* $\tilde{z}_{1j}$ *exponentially converges to zero for* $1 \le j \le m$.

In this step, it is unknown whether the leader's information is available to a quadrotor or not. However, with the aid of neighbors' information, the positions of all quadrotors converge to the desired position of the leader quadrotor if $\tilde{z}_{1j} = \alpha_{1j}$.

Step 3: Let $\bar{z}_{2j} = \tilde{z}_{2j} - \alpha_{1j}$ for $1 \le j \le m$, we have

$$\begin{aligned}\dot{\bar{z}}_{2j} &= -ge_3 + \beta_j f_j R_j e_3 + \phi_{1j}\hat{\theta}_{1j} + L_{2j}(v_j - z_{2j}) \\ &\quad + (p_j - z_{1j}) + \delta_{1j}\chi(e^{\lambda t}(v_j - z_{2j})) - \dot{\alpha}_{1j} - \ddot{h}_j \\ &\quad - \dot{z}_{20} + \ddot{h}_0 \end{aligned} \tag{22}$$

where $b_{ji} > 0$.

We assume that $f_j R_j e_3$ is a virtual control input and design it such that $\bar{z}_{2j}$ is bounded and converges to zero. In order to make the system (22) be the form of the systems in item 3 of Lemma 1, the virtual control for $f_j R_j e_3$ is chosen as

$$\begin{aligned}\alpha_{2j} &= ge_3 - \phi_{1j}\hat{\theta}_{1j} - L_{2j}(v_j - z_{2j}) - (p_j - z_{1j}) - \delta_j\chi(e^{\lambda t}(v_j - z_{2j})) + \dot{\alpha}_{1j} + \ddot{h}_j \\ &\quad - \sum_{i\in\mathcal{N}_j^a} b_{ji}(\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j, \quad \bar{z}_{20} = 0 \end{aligned} \tag{23}$$

$$\dot{\xi}_j = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\xi_j - \xi_i) - \rho_{1j}\chi\left(\sum_{i\in\mathcal{N}_j^a} b_{ji}(\xi_j - \xi_i)\right) \tag{24}$$

$$\dot{\rho}_{1j} = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\rho_{1j} - \rho_{1i}) \tag{25}$$

where $\xi_0 = \dot{z}_{20} - \ddot{h}_0$ and $\rho_{1,0} = \max_{t\in[0,\infty)} |\ddot{z}_{20}(t) - \dddot{h}_0|$.

For the systems in (24–25), by Lemma 1 (item 3 with $u_j = 0$) ρ_{1j} exponentially converges to $\rho_{1,0}$ and ξ_j exponentially converges to ξ_0 for $1 \le j \le m$.

With the aid of the virtual control α_{2j}, we have

$$\dot{\bar{z}}_{2j} = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j - \dot{z}_{20} + \ddot{h}_0 + \beta_j f_j R_j e_3 - \alpha_{2j} \tag{26}$$

With the aid of Lemma 1, the system with input $(\xi_1 - \dot{z}_{20} + \ddot{h}_0 + \beta_1 f_1 R_1 e_3 - \alpha_{21}, \ldots, \xi_m - \dot{z}_{20} + \beta_m f_m R_m e_3 - \alpha_{2m})$ and state $(\bar{z}_{21}, \ldots, \bar{z}_{2m})$ has ISS property. Since $\xi_j - \dot{z}_{20} + \ddot{h}_0$ exponentially converges to zero for $1 \le j \le m$, $\bar{z}_{2j}$ exponentially converges to zero if $\beta_j f_j R_j e_3 - \alpha_{1j}$ exponentially converges to zero for $1 \le j \le m$.

Step 4: We find f_j and the desired orientation R_j^d for j-th quadrotor. Let

$$f_j R_j^d e_3 = \alpha_{2j} \tag{27}$$

where $R_j^d = [b_{1j}^d, b_{2j}^d, b_{3j}^d]$, then

$$f_j = \|\alpha_{2j}\| \tag{28}$$

$$b_{3j}^d = \frac{\alpha_{2j}}{\|\alpha_{2j}\|}. \tag{29}$$

In (29), b_{3j}^d is not defined if $\alpha_{2j} = 0$. In this case, we define b_{3j}^d as follows

$$b_{3j}^d = \frac{\dot{\alpha}_{2j}}{\|\dot{\alpha}_{2j}\|}.$$

To define b_{1j}^d and b_{2j}^d, the information $b_{2,0}$ is required. However, $b_{2,0}$ is not available for all quadrotors. We propose the following distributed observer for j-th quadrotor.

$$\dot{r}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(r_j - r_i) - \rho_{2j}\chi\left(\sum_{i\in\mathcal{N}_j^e} a_{ji}(r_j - r_i)\right) \tag{30}$$

$$\dot{\rho}_{2j} = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\rho_{2j} - \rho_{2i}) \tag{31}$$

where $r_0 = b_{2,0}$ and $\rho_{20} = \max_{t\in[0,\infty)} |\dot{b}_{2,0}(t)|$.

For the systems in (30–31), under Assumption 1, by Lemma 1 $\lim_{t\to\infty}(r_j - b_{2,0}) \overset{exp.}{=} 0$ and $\lim_{t\to\infty}(\rho_{2j} - \rho_{2,0}) \overset{exp.}{=} 0$ for $1 \le j \le m$.

We choose

$$\bar{r}_j = r_j - r_j^\top b_{3j}^d b_{3j}^d \tag{32}$$

$$b_{2j}^d = \frac{\bar{r}_j}{\|\bar{r}_j\|} \tag{33}$$

$$b_{1j}^d = b_{2j}^d \times b_{3j}^d. \tag{34}$$

The desired attitude of R_j is chosen as

$$R_j^d = \left[b_{1j}^d, b_{2j}^d, b_{3j}^d\right] \tag{35}$$

and the desired quaternion $q_j^d = [\eta_j^d, (\epsilon_j^d)^\top]^\top$ of q_j is calculated by the equations (166–168) in [29] which are omitted here. The desired angular velocity is calculated by

$$\omega_j^d = 2A(q_j^d)^\top \frac{dq_j^d}{dt}. \tag{36}$$

Step 5: Since q_j is not a control input, q_j cannot be q_j^d. Let the difference between q_j and q_j^d be

$$\tilde{q}_j = (q_j^d)^{-1} \otimes q_j = [\tilde{\eta}_j, \tilde{\epsilon}_j^\top]^\top, \tag{37}$$

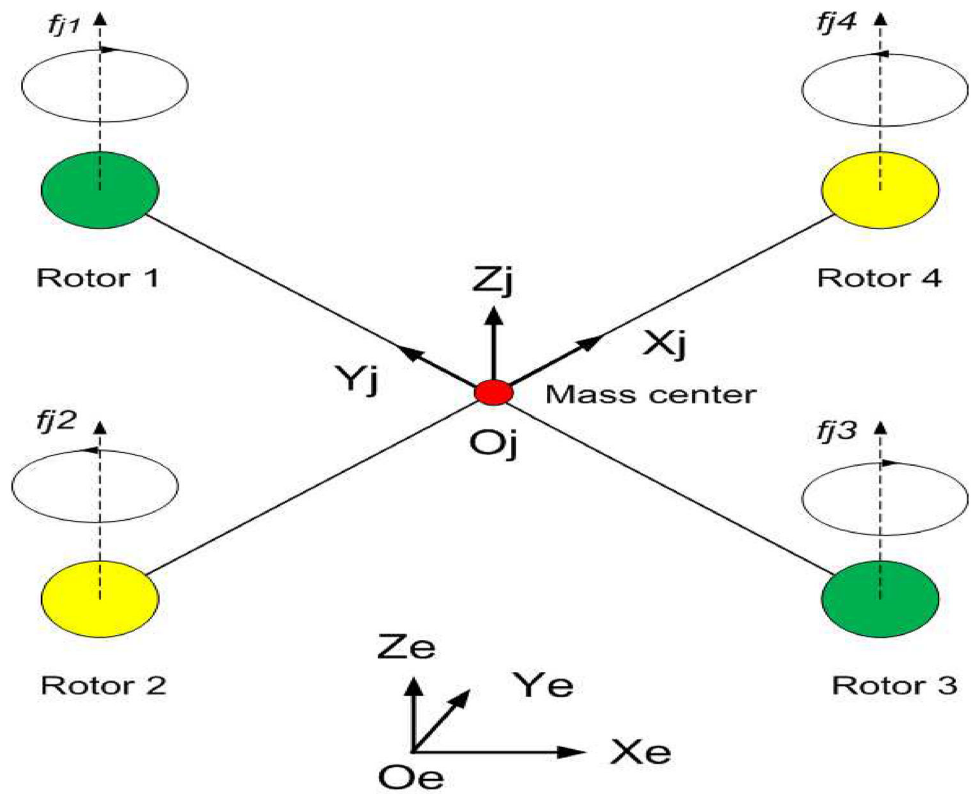

Fig. 1 Configuration of a quadrotor

Noting that

$$\begin{aligned} f_j R_j e_3 - \alpha_{2j} &= f_j R_j e_3 - f_j R_j^d e_3 = f_j R_j^d (\tilde{R}_j - I) e_3 \\ &= f_j R_j^d (\tilde{\eta} I_3 + S(\tilde{\epsilon}_j)) S(e_3) \tilde{\epsilon}_j \end{aligned}$$

we have

$$\begin{aligned} \dot{\bar{z}}_{2j} = &- \sum_{i \in \mathcal{N}_j^a} b_{ji} (\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j - \dot{z}_{20} + f_j R_j^d (\tilde{\eta} I_3 \\ &+ S(\tilde{\epsilon}_j)) S(e_3). \end{aligned} \tag{38}$$

For the systems in (38), the following results can be proved with the aid of Lemma 1 and the proof is omitted.

Lemma 4 *For the systems in (38), under Assumption 1, if $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \leq j \leq m$, then $\bar{z}_{2j}$ exponentially converges to zero for $1 \leq j \leq m$.*

The derivative of $\tilde{q}_j$ is

$$\dot{\tilde{q}}_j = \frac{1}{2} A(\tilde{q}_j)(\omega_j - \tilde{R}_j^\top \omega_j^d) \tag{39}$$

where $\tilde{R}_j = (R_j^d)^\top R_j$.

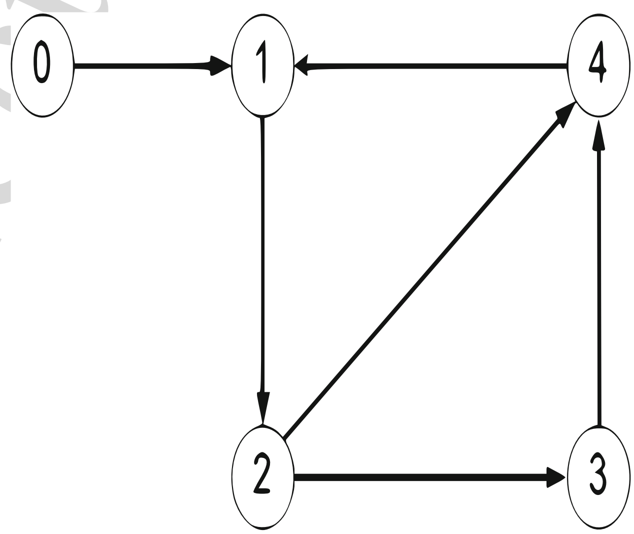

Fig. 2 Communication graph between VTOL vehicles

We choose a Lyapunov function candidate

$$\begin{aligned} V_{2j} = 2(1 - \tilde{\eta}_j) = \tilde{\epsilon}_j^\top \tilde{\epsilon}_j \\ +(1 - \tilde{\eta}_j)^2 \end{aligned} \tag{40}$$

The derivative of V_{2j} is

$$\dot{V}_{2j} = \tilde{\epsilon}_j^\top (\omega_j - \tilde{R}_j^\top \omega_j^d)$$

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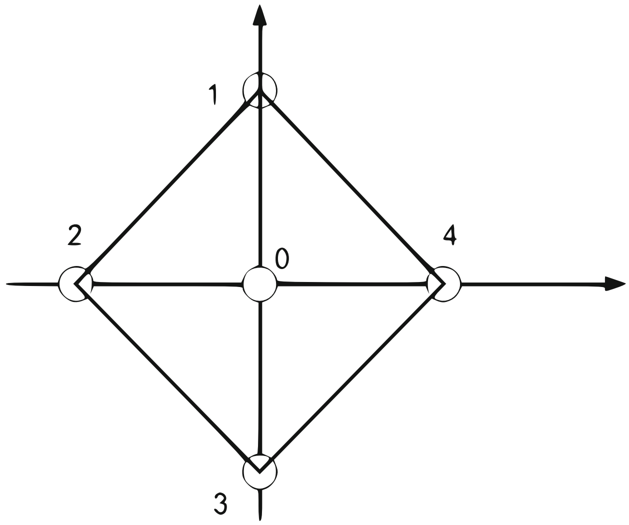

Fig. 3 Desired formation

To make $\tilde{q}_j$ converge to an identity quaternion, a virtual controller μ_j for ω_j can be chosen as

$$\mu_j = -k_{1j}\tilde{\epsilon}_j + \tilde{R}_j^\top \omega_j^d \tag{41}$$

where k_{1j} is a positive constant. Then,

$$\dot{V}_{2j} = -k_{1j}\tilde{\epsilon}_j^\top \tilde{\epsilon}_j + \tilde{\epsilon}_j^\top(\omega_j - \mu_j).$$

Step 6: Since ω_j is not a real control input, one cannot let ω_j be μ_j. Define

$$\tilde{\omega}_j = \omega_j - \mu_j,$$

then,

$$\dot{\tilde{q}}_j = \frac{1}{2}A(\tilde{\eta}_j, \tilde{\epsilon}_j)(-k_3\tilde{\epsilon}_j + \tilde{\omega}_j) \tag{42}$$

$$\begin{aligned} J_j\dot{\tilde{\omega}}_j &= S(J_j\omega_j)\omega_j + \tau_j - J_j\dot{\mu}_j + d_{2j} \\ &= \tau_j - (S(\omega_j)\Gamma(\omega_j) + \Gamma(\dot{\mu}_j))a_j + d_{2j} \end{aligned} \tag{43}$$

where $\Gamma(\omega_j) = \mathrm{diag}([\omega_j^\top, \omega_j^\top, \omega_j^\top])$ and $a_j = [J_j^1, J_j^2, J_j^3]^\top$ where J_j^i is the i-th row of J_j.

A neural network is applied to approximate d_{2j}. Let ϕ_{2j} be a collection of basis vectors, there exists an optimal vector θ_{2j} such that

$$d_{2j} = \phi_{2j}\theta_{2j} + \epsilon_{2j} \tag{44}$$

where ϵ_{2j} is the approximation error vector and $\|\epsilon_{2j}\| \le \delta_{2j}$.

To design a control law such that (5–6) are satisfied, we choose a Lyapunov function candidate

$$\begin{aligned} V_{3j} = {}& V_{2j} + \frac{1}{2}e^{2\lambda t}\tilde{\omega}_j^\top J_j\tilde{\omega}_j + \frac{\gamma_{3j}^{-1}}{2}(a_j - \hat{a}_j)^\top(a_j - \hat{a}_j) \\ & + \frac{\gamma_{4j}^{-1}}{2}(\theta_{2j} - \hat{\theta}_{2j})^\top(\theta_{2j} - \hat{\theta}_{2j}) \end{aligned}$$

where γ_{3j} and γ_{4j} are positive constants, $\hat{a}_j$ is an estimate of a_j, and $\hat{\theta}_{2j}$ is an estimate of θ_{2j}. The derivative of V_{3j} is

$$\begin{aligned} \dot{V}_{3j} = {}& -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j + \tilde{\epsilon}_j^\top\tilde{\omega}_j + e^{2\lambda t}\tilde{\omega}_j^\top(\tau_j - (S(\omega_j)\Gamma(\omega_j) \\ & + \Gamma(\dot{\mu}_j - \lambda\tilde{\omega}_j))a_j + \phi_{2j}\theta_{2j} + \epsilon_{2j}) \\ & - \gamma_{3j}^{-1}(a_j - \hat{a}_j)^\top\dot{\hat{a}}_j - \gamma_{4j}^{-1}(\theta_{2j} - \hat{\theta}_{2j})^\top\dot{\hat{\theta}}_{2j}. \end{aligned}$$

The control law τ_j and the update laws $\hat{a}_j$ and $\hat{\theta}_{2j}$ are chosen as follows:

$$\begin{aligned} \tau_j = {}& -k_{2j}\tilde{\omega}_j - e^{-2\lambda t}\tilde{\epsilon}_j + (S(\omega_j)\Gamma(\omega_j) \\ & + \Gamma(\dot{\mu}_j) - \lambda\tilde{\omega}_j)\hat{a}_j - \phi_{2j}\hat{\theta}_{2j} - \delta_{2j}\chi(e^{\lambda t}\tilde{\omega}_j) \end{aligned} \tag{45}$$

$$\dot{\hat{a}}_j = -\gamma_{3j}e^{2\lambda t}(S(\omega_j)\Gamma(\omega_j) + \Gamma(\dot{\mu}_j) - \lambda\tilde{\omega}_j)^\top\tilde{\omega}_j \tag{46}$$

$$\dot{\hat{\theta}}_{2j} = \gamma_{4j}e^{2\lambda t}\phi_{2j}^\top\tilde{\omega}_j \tag{47}$$

where k_{2j} is a positive constant. Then,

$$\begin{aligned} \dot{V}_{3j} = {}& -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j - k_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\tilde{\omega}_j - \delta_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\chi(e^{\lambda t}\tilde{\omega}_j) \\ & + e^{\lambda t}\tilde{\omega}_j^\top\epsilon_{2j} \\ \le {}& -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j - k_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\tilde{\omega}_j + 3\delta_{2j}e^{-(\kappa-\lambda)t}. \end{aligned} \tag{48}$$

Based on the above controller design procedure, we have the following results.

Theorem 1 *For a leader quadrotor and m follower quadrotors in (1–4), it is given a desired formation defined by h_j for $0 \le j \le m$. Under Assumptions 1–3, the distributed control inputs (f_j, τ_j) in (28) and (45) ensure that (5–6) are satisfied and $(\beta_j, \hat{a}_j, \hat{\theta}_{1j}, \hat{\theta}_{2j})$ are bounded.*

Proof Integrating both sides of (48), it can be shown that $V_{3j} \in \mathcal{L}_\infty$. So, $\tilde{\epsilon}_j \in \mathcal{L}_\infty$, $e^{\lambda t}\tilde{\omega}_j \in \mathcal{L}_\infty$, $\hat{a}_j \in \mathcal{L}_\infty$, and $\hat{\theta}_j \in \mathcal{L}_\infty$. So, $\tilde{\omega}_j$ is bounded and exponentially converges to zero. Integrating both sides of (48), it can also be shown that $\tilde{\epsilon}_j \in \mathcal{L}_2$. By Lemma 1 in [30], $\tilde{\epsilon}_j$ converges to zero. So, $\tilde{q}_j$ converges to an identity quaternion for $1 \le j \le m$.

Next, we show $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \le j \le m$. With the aid of V_{2j}, we have

$$\begin{aligned} \dot{V}_{2j} &\le -\frac{k_{1j}}{2}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2 \\ &= -\frac{k_{1j}}{2}V_{2j} + \frac{k_{1j}}{8}V_{2j}^2 + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2 \end{aligned}$$

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Fig. 4 Time response of $p_j - z_{1j}$ for $1 \leq j \leq 4$

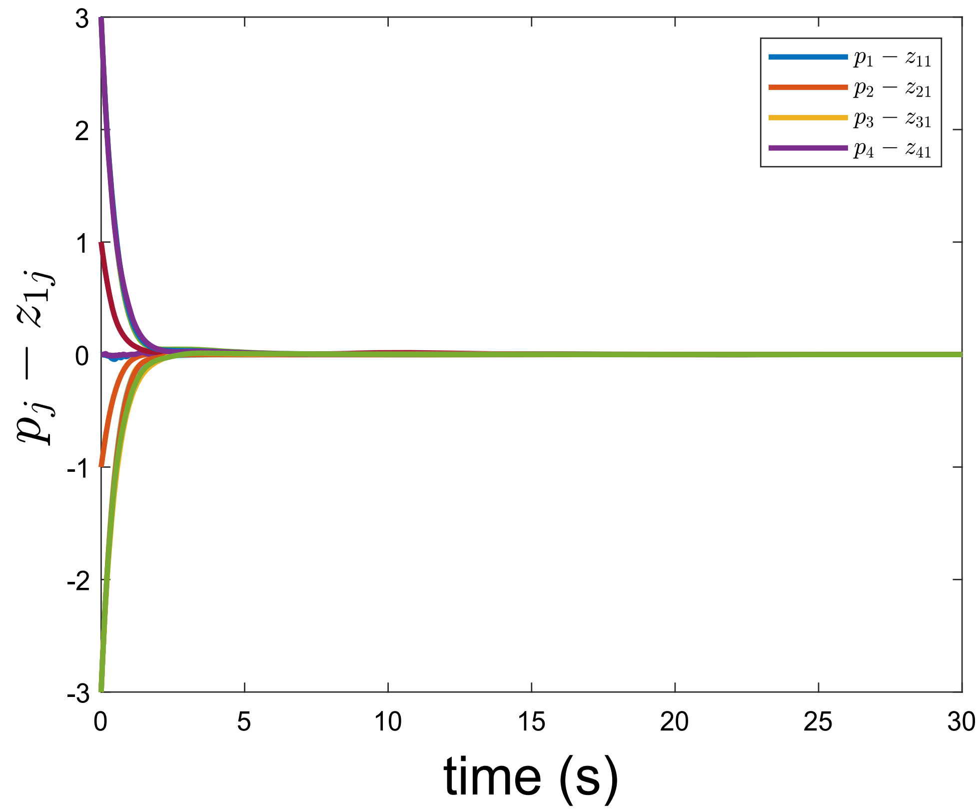

Fig. 5 Time response of $v_j - z_{2j}$ for $1 \leq j \leq 4$

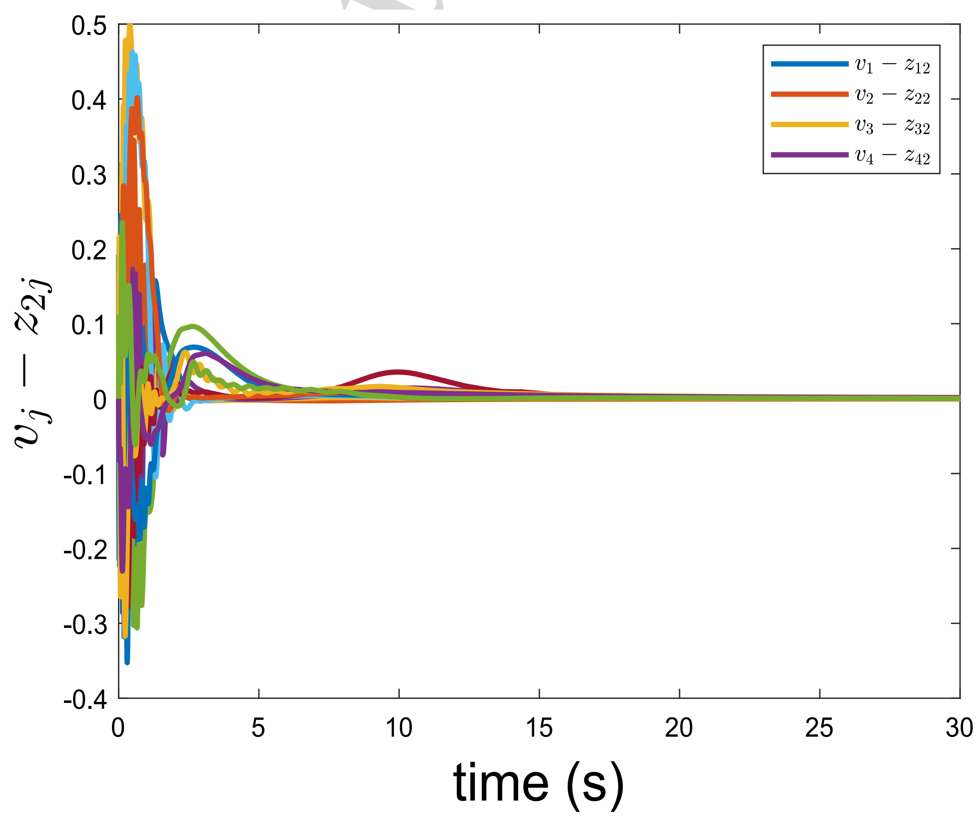

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Fig. 6 Time response of $p_j - h_j - (p_0 - h_0)$ for $1 \le j \le 4$

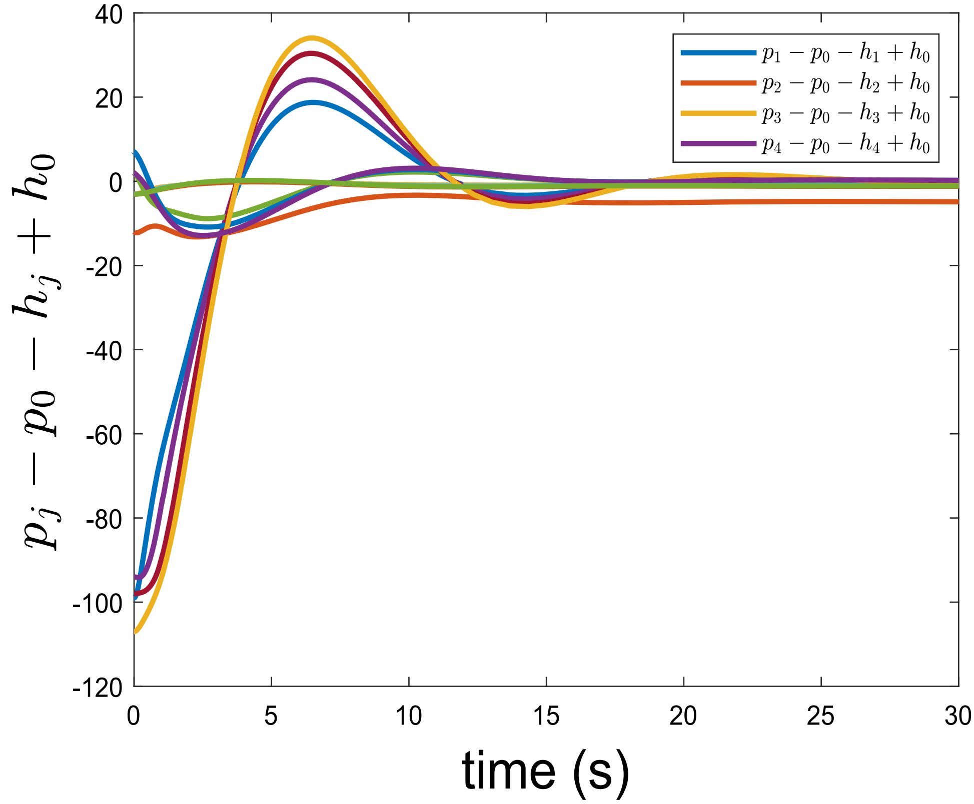

Fig. 7 Time response of $b_{2j} - b_{20}$ for $1 \le j \le 4$

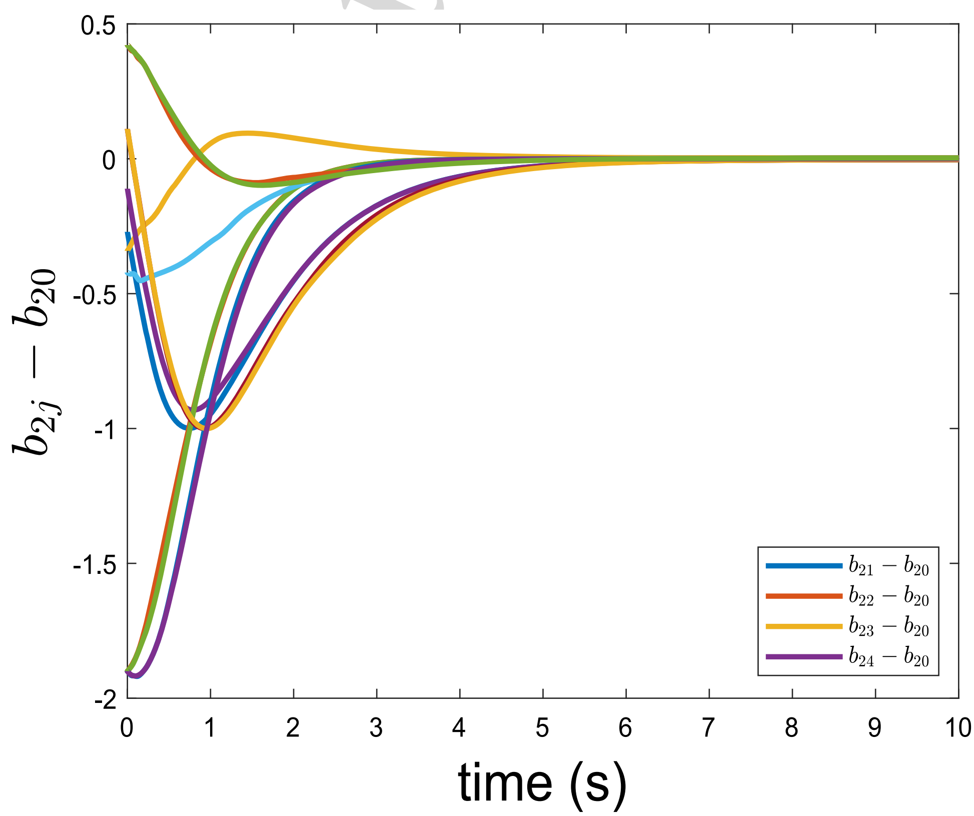

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Fig. 8 Time response of β_j for $1 \leq j \leq 4$

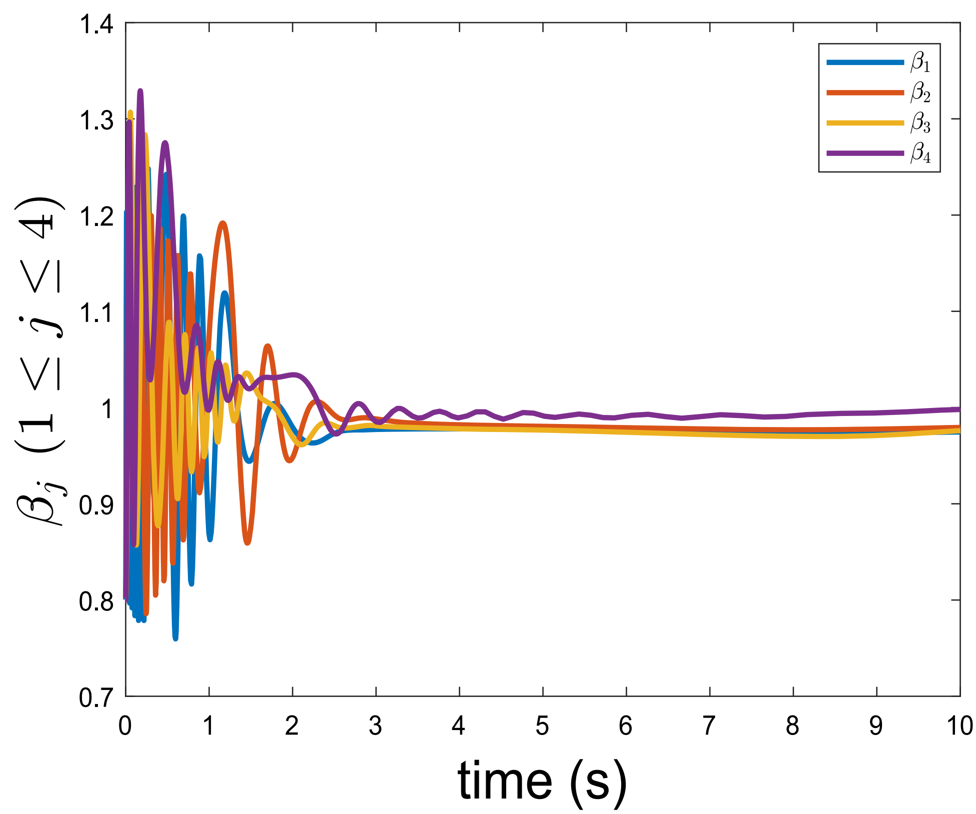
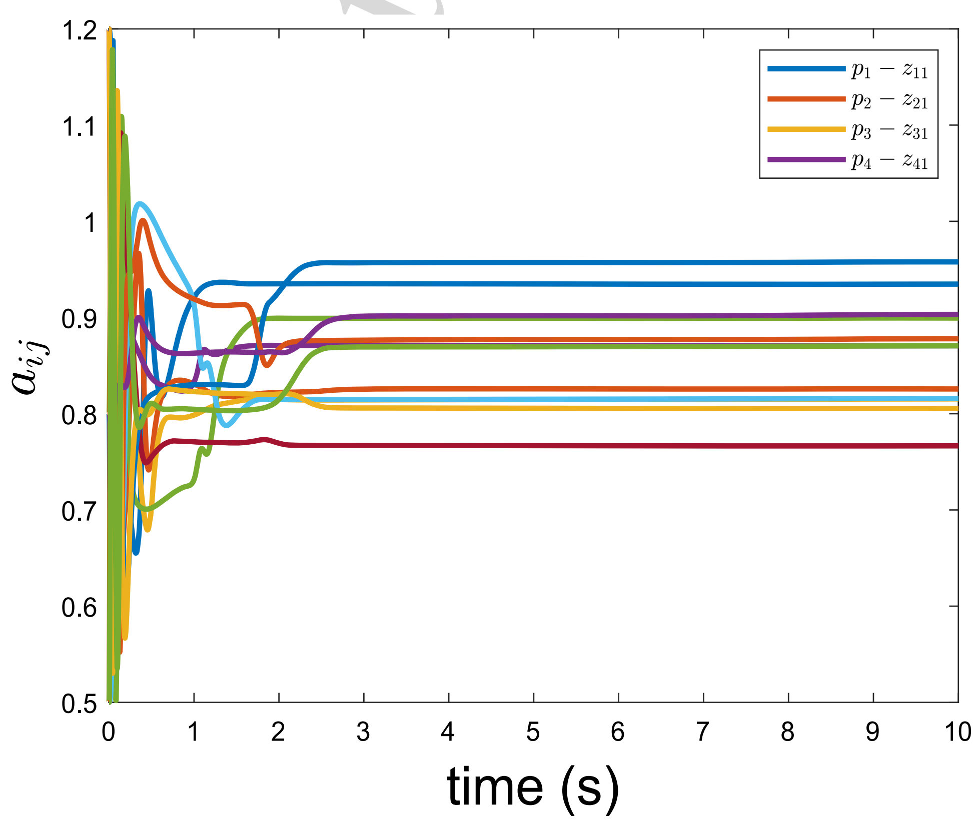


Fig. 9 Time response of a_{ij} for $1 \leq j \leq 4$ and $1 \leq i \leq 3$



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Since $\tilde{\eta}_j$ converges to one, there exists a finite time T such that V_{2j} is less than one. After time T,

$$\dot{V}_{2j} \leq -\frac{k_{1j}}{2}V_{2j} + \frac{k_{1j}}{8}V_{2j} + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2$$
$$= -\frac{3k_{1j}}{8}V_{2j} + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2$$

Noting $\tilde{\omega}_j$ exponentially converges to zero, it can be shown that V_{2j} exponentially converges to zero with the aid of the comparison lemma in [26]. So, $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \leq j \leq m$ after a finite time. With the aid of Lemma 4, $\bar{z}_{2j}$ exponentially converges to zero after a finite time for $1 \leq j \leq m$. By Lemma 3, $\tilde{z}_{1j}$ exponentially converges to zero after a finite time for $1 \leq j \leq m$. With the aid of the definition of $\tilde{z}_{1j}$, (18) holds. So, (5) is satisfied. (6) is satisfied because $\tilde{q}_j$ exponentially converges to an identity quaternion after a finite time. □

In the controller design procedure, the uncertainty in the dynamics of each quadrotor and the uncertain knowledge of the leader's information are dealt with in different steps. We call this design procedure the uncertainty decomposition approach.

In Theorem 1, in order to make the quadrotors come into the desired formation exponentially the control laws and the adaptive update laws contain the term $e^{\lambda t}$. If we choose a weight function carefully, it is possible to make the quadrotors come into the desired formation within a finite time. They are omitted due to space limitation.

4 Simulation results

The proposed results can be applied to design distributed controllers for formation flying of multiple quadrotors. Considered five quadrotors. The configuration of each quadrotor is shown in Fig. 1. The dynamics of quadrotor j can be written as (1–4), where the total thrust f_j and the generalized moment vector τ_j are generated by the four rotors. For simplicity, we ignore the dynamics of each rotor and consider f_j and τ_j as control inputs. In the simulation, it is assumed that $m_j = 1$kg and inertia tensor $J_j = \mathrm{diag}([1, 1, 1])$kg m^2. In the controller design, m_j and J_j are not exactly known. However, it is known that $m_j \in [0.8, 1.2]$kg, i.e., $\bar{m} = 1.2$kg and $\underline{m} = 0.8$kg.

In the simulation, it is assumed that the trajectory p_0 and $b_{2,0}$ of the leader VTOL vehicle are

$$p_0(t) = \left[\, 100\cos\tfrac{t}{20}, 100\sin\tfrac{t}{20}, 10 - 10\exp(-0.1t) \,\right]^\top$$
$$b_{2,0} = \left[\sin\frac{\pi t}{360}, \cos\frac{\pi t}{360}, 0\right]^\top.$$

The communication directed graph is shown in Fig. 2. It is can be verified that node 0 is globally reachable.

The desired formation for quadrotors is shown in Fig. 3, where $h_0 = [0, 0, 0]^\top$, $h_1 = [0, 5, 0]^\top$, $h_2 = [-5, 0, 0]^\top$, $h_3 = [0, -5, 0]^\top$, and $h_4 = [5, 0, 0]^\top$.

Distributed control laws can be designed with the aid of the procedure in the last section. The simulation was done for one group of control parameters. Figures 4, 5 show the time response of $p_j - z_{1j}$ and $v_j - z_{2j}$, respectively. It is shown that z_{1j} and z_{2j} are good estimates of p_j and v_j, respectively. Figure 6 shows the time response of $p_j - h_j - (p_0 - h_0)$ for $1 \leq j \leq 4$. It is shown that the VTOL vehicles come into the desired formation and follow the leader quadrotor. The time response of $b_{2j} - b_{20}$ is shown in Fig. 7, which shows that the Y-axis of the body frame of each quadrotor converges to the desired direction. The estimate β_j of $1/m_j$ for $1 \leq qj \leq 4$ are shown in Fig. 8. The time response of $\hat{a}_{ij}$ for $1 \leq j \leq 4$ and $1 \leq i \leq 3$ are shown in Fig. 9. They are bounded and confirm the claims in the theorem. The above simulation results show the effectiveness of the results in Theorem 1.

5 Conclusion

This paper considered the formation flying of multiple quadrotors with a desired attitude in the presence of parametric and nonparametric uncertainty. With the aid of the backstepping technique, a multi-step controller design approach has been proposed. With the aid of the proposed approach, distributed robust adaptive controllers were proposed such that the formation tracking errors and the attitude tracking errors exponentially converge to zero. Simulation results show the effectiveness of the proposed controllers for formation flying of five quadrotors.

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Author contributions The first author did the simulation. The main results were obtained by the second author.

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Declarations

Competing interest The authors declare no competing interests.

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