Pure Mathematics Seminar

Linear Decision Trees, Subspace Arrangements, and Discrete Morse Theory

Dr. Jacob White
SMSS, UTRGV

A linear decision tree is a computational model for determining membership in a subset $X \subseteq \mathbb{R}^n$. One motivating example is the $k$-equal arrangement, which consists of all $n$-tuples that have at least $k$ equal coordinates.

In 1995, Björner and Lovász gave a topological lower bound for the number of leaves in a linear decision tree to determine membership in a union of affine subspaces $\mathcal{A}$. They showed that the number of leaves is at least $\sum_{i=0}^{\infty} \beta_i(\mathbb{R}^n \setminus \mathcal{A})$. Then Björner and Welker computed the Betti numbers for the $k$-equal arrangement. These results spurred significant research activity into the study of subspace arrangements.

We give a new proof of the Björner-Lovász theorem, using discrete Morse theory. We show that, given a linear decision tree $T$ for a subspace arrangement $\mathcal{A}$, there exists a cellular model for $\mathbb{R}^n \setminus \mathcal{A}$ whose cells are indexed by the NO-leaves of $T$. This implies new lower bounds for decision tree complexity which take into account torsion elements in $\mathbb{R}^n \setminus \mathcal{A}$, and not just the Betti numbers. We also give at least one new example of a ‘natural’ decision problem for which torsion arises.

Date: Wednesday, October 31, 2018
Time: 12:15 pm
Place: Edinburg: EMAGC 1.410, Brownsville: BLIBR 2.206

The talk will delivered live at the Edinburg campus and will be streamed to the Brownsville campus

Coffee might be served.

For further information or for special accommodations, please contact Dr. Sergey Grigorian via email at sergey.grigorian@utrgv.edu, or Dr. Alexey Garber at alexey.garber@utrgv.edu, or visit the webpage http://www.utrgv.edu/math/news-events/seminars/puremath/index.htm