Schedule of talks.

Monday (May 1)
Morning session

9:30-10:15

Thomas Hales (University of Pittsburgh)

Bad packings: the Reinhardt conjecture as an optimal control problem

Abstract. It is easier to pack some shapes in the plane than others. For example, squares tile the plane with no wasted space, but even the best packing of identical circular disks leaves about 10% of the plane unfilled.

This talk will discuss Reinhardt's problem, which asks for the worst possible centrally symmetric convex shape for packing. In 1934, Reinhardt conjectured that the worst shape is an octagon with smoothed corners. We show how to formulate the problem as an optimal control problem. Optimal control theory gives us a collection of tools that we hope will lead to a solution to the problem.

Here is a blog post by John Baez about the problem: `http://blogs.ams.org/visualinsight/2014/11/01/packing-smoothed-octagons/`. My recent work is described in a blog post `https://jiggerwit.wordpress.com/2017/03/05/bad-packings/` and preprint `https://arxiv.org/abs/1703.01352`.

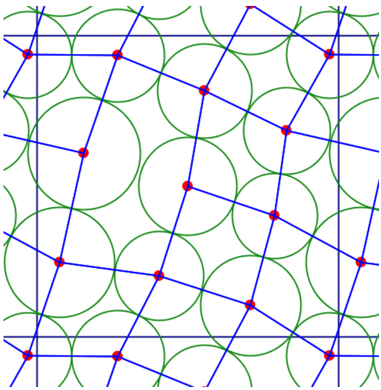
Monday (May 1)
Morning session

10:30-11:15

Robert Connelly (Cornell University)

The Isostatic Conjecture

Abstract. We show that a jammed packing of disks with generic radii, in a generic container, is such that the minimal number of contacts occurs and there is only one dimension of equilibrium stresses. We also point out some connections to packings with different radii and results in the theory of circle packings whose graph forms a triangulation of a given topological surface. The following is an example of an isostatic packing of 10 disks in a torus given by square lattice.



This is a joint work with Evan Solomonides and Maria Yampolskaya (Cornell University).

Monday (May 1)
Morning session

11:30-12:15

Senya Shlosman (Institute for Information Transmission Problems, Russia)

Kissing spheres – a personal story

Abstract. I will describe some properties of the 24-dimensional "manifold" of configurations of 12 nonoverlapping equal spheres of radius r touching a central sphere of radius 1. In particular, I will explain the structure of this manifold in the vicinity of its singular points. The talk is based on the paper by Rob Kusner, Wöden Kusner, Jeffrey C. Lagarias and myself: *The twelve spheres problem* `arXiv:1611.10297` as well as on a work in progress, by the same authors.

Monday (May 1)
Afternoon session

2:30-3:15

Peter Dragnev (Indiana University – Purdue University Fort Wayne)

Logarithmic and Riesz Equilibrium for Multiple Sources on the Sphere – the Exceptional Case

Abstract. In this talk we consider the minimal discrete and continuous energy problems on the unit sphere $\mathbb{S}^d$ in the Euclidean space $\mathbb{R}^{d+1}$ in the presence of an external field due to finitely many localized charge distributions on $\mathbb{S}^d$, where the energy arises from the Riesz potential $1/r^s$ (r is the Euclidean distance) for the critical Riesz parameter $s = d - 2$ if $d \geq 3$ and the logarithmic potential $\log(1/r)$ if $d = 2$. Individually, a localized charge distribution is either a point charge or assumed to be rotationally symmetric. The extremal measure solving the continuous external field problem for weak fields is shown to be the uniform measure on the sphere but restricted to the exterior of spherical caps surrounding the localized charge distributions. The radii are determined by the relative strengths of the generating charges. Furthermore, we show that the minimal energy points solving the related discrete external field problem are confined to this support. For $d-2 \leq s < d$, we show that for point sources on the sphere, the equilibrium measure has support in the complement of the union of specified spherical caps about the sources. Numerical examples are presented to illustrate our results.

Joint work with: Johann Brauchart - TU Graz, Ed Saff - Vanderbilt, Robert Womersley - UNSW.

Monday (May 1)
Afternoon session

3:30-4:15

Peter Boyvalenkov (Institute of Mathematics and Informatics, Bulgarian Academy of Sciences)

On a duality in computing bounds for spherical codes

Abstract. We introduce and discuss a duality in computation of universal lower bounds for potential energy and universal upper bounds for cardinality of spherical codes. The gluing point is ensured by a kind of quadrature formulas first applied in the field by Levenshtein. In one direction, we show how a program, called SCOD, which was developed for obtaining linear programming (LP) bounds for spherical codes of prescribed dimension and minimum distance can be updated and utilized to produce data for obtaining good LP bounds for energy of spherical codes of prescribed dimension and cardinality and for fixed potential function which is absolute monotone. Dually in a sense, we show how recent advance in bounding energies can serve data for obtaining good LP bounds for maximal codes.

This is joint work with P. Dragnev, D. Hardin, P. Kazakov, E. Saff, M. Stoyanova.

Monday (May 1)
Afternoon session

4:30-5:15

Doug Hardin (Vanderbilt University)

Universal lower bounds for the energy of spherical codes: lifting the Levenshtein framework

Abstract. For a potential h defined on the interval $[-1, 1)$ and a finite point configuration (or *code*) C on the unit sphere $\mathbb{S}^{n-1}$, the h-energy of C is given by

$$E(C, n, h) := \sum_{x \neq y \in C} h(\langle x, y \rangle).$$

H. Cohn and A. Kumar proved that every sharp spherical code (a code with m distinct inner products that is also a spherical design of strength $2m - 1$) is universally optimal; i.e., such a code minimizes the potential h-energy among all codes of cardinality $|C|$ for any absolutely monotone potential h. In addition, they proved that the 600-cell (a code with 120 points on $\mathbb{S}^3$) is also a universally optimal code.

In this talk we present lower bounds for energy of the form

$$E(C, n, h) \geq N^2 \sum_{i=1}^{m} \rho_i h(\alpha_i),$$

where the nodes $\{\alpha_i\}$ and weights $\{\rho_i\}$ depend only on the cardinality N and dimension n and are obtained from a quadrature rule framework developed by Levenshtein in relation to maximal codes. The Hermite interpolation polynomials to h at $\{\alpha_i\}$ solve a Delsarte-Yudin linear program for polynomials of degree at most $2m - 2$ or

$2m-1$ depending on whether -1 is among the nodes. We extend these bounds by considering the linear program for polynomials of higher degree, essentially lifting the Levenshtein framework and apply these techniques to obtain universal lower bounds for the interesting cases of $N = 24$ and 120 points on $\mathbb{S}^3$. In the case $N = 24$ we obtain the best possible linear programming bound and in the case $N = 120$ we obtain a simplified and (somewhat) systematic proof of the universal optimality of the 600-cell.

Joint work with Peter Boyvalenkov (Bulgarian Academy of Sciences), Peter Dragnev (Indiana-Purdue University Fort Wayne) Ed Saff (Vanderbilt University), and Maya Stoyanova (Sofia University).

Tuesday (May 2)
Morning session

9:30-10:15

Marjorie Senechal (Smith College)

Icosahedral Snowflakes?

Abstract. Long before the discovery of H, O, and H_2O, Johannes Kepler proposed, incorrectly but astutely, that hexagonal snowflakes grow by the accretion of invisible spherical particles in a densely packed array. And so the science of crystallography was born. Half a milennium later, once-thought-to-be-impossible icosahedral crystals again raised the question of growth and form, but for these crystals their relation remains murky.

Two models, decorated tilings and nested clusters, have been used to describe the arrangements of atoms in icosahedral crystals. But both models have trouble with growth. In this talk I will discuss a particular case, the Yterrbium-Cadmium icosahedral crystal and its close periodic relatives, and show how a modified cluster model may show us a way out.

Joint work with Jean E. Taylor, Erin G. Teich, Pablo Damasceno, Yoav Kallus.

Tuesday (May 2)
Morning session

10:30-11:15

Boris Bukh (Carnegie Mellon University)

One-sided epsilon-approximants

Abstract. Two common approximation notions in discrete geometry are ε-nets and ε-approximants. Of the two, ε-approximants are stronger. For the family of convex sets, small ε-nets exist while small ε-approximants unfortunately do not. In this talk, we introduce a new notion "one-sided ε-approximants", which is of intermediate strength, and prove that small one-sided ε-approximants do exist. The proof is based on a (modification of) the regularity lemma for words by Axenovich–Person–Puzynina. Joint work with Gabriel Nivasch.

Tuesday (May 2)
Morning session

11:30-12:15

Alan Haynes (University of Houston)

Lower bounds for numbers of distinct frequencies of patterns in cut and project sets

Abstract. In this talk we investigate the problem of understanding, for $r > 1$, the numbers of distinct frequencies of patterns of size r which are possible in d dimensional cut and project sets constructed from $d + 1$ dimensional total spaces. For $d = 1$ this is a problem about Sturmian sequences, and it is a corollary of results of Morse and Hedlund (1930's and 40's), together with the well known Steinhaus theorem (1950's), that the number of distinct frequencies is always at most 3, regardless of what r is. For $d > 1$ the problem depends in part on the shape of the patterns being considered, and it is considerably more difficult. Our main result (joint work with Jens Marklof) is that, for any shape of pattern, and for almost every d dimensional physical space in $\mathbb{R}^{d+1}$, the number of distinct frequencies of patterns of size r is not bounded, as $r \to \infty$. As we will see, this problem is closely related to the Littlewood conjecture in Diophantine approximation. Its solution, which answers a question posed (in a slightly different setting) by Erdős, Geelen, and Simpson, ultimately relies on results from ergodic theory in the space of unimodular lattices in $\mathbb{R}^{d+1}$.

Tuesday (May 2)
Afternoon session

2:30-3:15

Włodzimierz Kuperberg (Auburn University)

Unavoidable crossings in finite coverings

Abstract. Two convex disks in the plane are said to *cross each other* if the removal of their intersection causes each disk to fall into disjoint components. The possibility of crossings in (infinite) coverings of the plane with congruent copies of a convex disk presents an obstacle in proving L. Fejes Tóth's conjecture (1950) about the thinnest coverings, analogous to a corresponding theorem for packings. A partially analogous version of the theorem requires the covering to be crossing-free. For about 60 years, it was commonly believed that crossings in a plane covering by congruent convex disks, appear to be counterproductive for producing low density, hence are always avoidable. The first example that illustrated the difficulties in attempts to remove crossings by local, finite rearrangements of the disks was given by A. Heppes (unpublished), followed soon by a more general construction by G. Wegner (1980).

In 2010, we produced an example of a convex pentagon P such that in any minimum density covering of the plane with congruent copies of P crossings must occur. Thus any attempt of proving the conjecture of L. Fejes Tóth must take into account the possibility of unavoidable crossings.

In this talk we consider finite versions of the phenomenon of unavoidable crossings in coverings, in the same vein as the constructions of Heppes and Wegner, and we present a variety of new examples of a similar nature, with short and elementary proofs.

This is a joint work with András Bezdek.

Tuesday (May 2)
Afternoon session

3:30-4:15

Antoine Deza (McMaster University)

Lattice polytopes with large diameter and many vertices

Abstract. A lattice (d, k)-polytope is the convex hull of a set of points in dimension d whose coordinates are integers between 0 and k. In this talk, we will introduce lattice polytopes generated by the primitive vectors of bounded norm. These primitive zonotopes can be seen as a generalization of the permutahedron of type B_d. We will highlight connections between the primitive zonotopes and the largest possible diameter of lattice (d, k)-polytopes, and between the computational complexity of multicriteria matroid optimization. Tightening of the bounds for the largest possible diameter of a lattice (d, k)-polytope, complexity results, conjectures, and open questions will be discussed. Based on joint works with Nathan Chadder, George Manoussakis, Shmuel Onn, and Lionel Pournin.

Tuesday (May 2)
Afternoon session

4:30-5:15

Anthony Harrison (Kent State University)

The lattice size of lattice polygons with respect to the 2-simplex

Abstract. The lattice size of a lattice polygon P, denoted $\mathrm{ls}(P)$, is the smallest number n such that the image of P under an affine unimodular transformation is contained within the n-dilate of the standard 2-simplex. An optimal transformation T, one such that TP fits in the smallest possible dilate, can be used to find a "better" parametrization of a toric surface. Results from Castryck, Cools, and Shicho show that there is a recursive algorithm to find such a T by relating $\mathrm{ls}(P)$ to the lattice size of the convex hull of the interior lattice points of P. We have developed an algorithm that needs only the vertices of P and so avoids the computational expense of determining the interior lattice points. We show that if a fixed, finite set of transformations does not yield a "smaller" image of P, then P can be translated to fit in the smallest possible dilate of the simplex.

Wednesday (May 3)
Morning session

9:30-10:15

Alexander Dranishnikov (University of Florida)

On topological complexity of robot motion

Abstract. Topological complexity $TC(X)$ of a configuration space X was defined by Farber as numerical invariant that measures the navigational complexity of X. The invariant TC is similar to the Lusternick-Schnirelman category of a space $catX$. We will discuss the problem of computation of $TC(X)$ as well as the relation of TC to cat.

Wednesday (May 3)
Morning session

10:30-11:15

Boris Okun (University of Wisconsin–Milwaukee)

Action Dimension and L^2-homology

Abstract. The action dimension of a group G, $\operatorname{actdim}(G)$ is the least dimension of a contractible manifold which admits a proper G-action. The action dimension conjecture states that L^2-homology of any group G vanishes above $\operatorname{actdim}(G)/2$.

I will explain the equivalence of this conjecture to the classical Singer conjecture, and describe its current status.

This is based on a joint work with Kevin Schreve.

Wednesday (May 3)
Morning session

11:30-12:15

Boris N. Apanasov (University of Oklahoma)

Block-Building & Deformations of Polyhedra, Group Homomorphisms and Quasiregular Mappings

Abstract. We discuss our method of block-building of conformal polyhedra in the 3-sphere, their "bending" deformations and several applications to geometry, topology and geometric analysis. This is related to varieties of conformal structures on closed hyperbolic 3-manifolds, to the shape of non-trivial compact 4-dimensional cobordisms M (cf. [1], [4]) whose interiors have complete hyperbolic structures (how the global geometry and topology of such cobordisms depends on properties of the variety of discrete representations of the fundamental group of its boundary ∂M -cf. [2, 3]), to different ergodic actions of a uniform hyperbolic 3-lattice [6], as well as to M.A.Lavrentiev problem on locally homeomorphic quasi-regular mappings in 3-space [7]. This gives also a new view on Andreev's hyperbolic polyhedron theorem.

References

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[3] Boris Apanasov, *Conformal geometry of discrete groups and manifolds.* - De Gruyter Expositions in Math. **32**, W. de Gruyter, Berlin - New York, 2000, XIV + 523 pp.

[4] Boris Apanasov, *Quasisymmetric embeddings of a closed ball inextensible in neighborhoods of any boundary points*, Ann. Acad. Sci. Fenn., Ser. AI **14:2** (1989), 243-255.

[5] Boris Apanasov, *Hyperbolic 4-cobordisms and group homomorphisms with infinite kernel.* - Atti Semin. Mat. Fis. Univ. Modena Reggio Emilia, **57**, 2010, 31-44.

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[7] Boris Apanasov, *Topological barriers for locally homeomorphic quasiregular mappings in 3-space.* - to apppear, http://arxiv.org/abs/1510.08951.

Wednesday (May 3)
Afternoon session

2:30-3:15

Imre Bárány (Alfréd Rényi Institute of Mathematics & University College London)

Tverberg plus minus

Abstract. We prove a Tverberg type theorem: Given a set $A \subset \mathbb{R}^d$ in general position with $|A| = (r-1)(d+1)+1$ and $k \in \{0, 1, \dots, r-1\}$, there is a partition of A into r sets $A_1, \dots, A_r$ (where $|A_p| \leq d+1$ for each p) with the following property. The unique $z \in \bigcap_{p=1}^{r} \operatorname{aff} A_p$ can be written as an affine combination of the elements in A_p: $z = \sum_{x \in A_p} \alpha(x)x$ for every p and exactly k of the coefficients $\alpha(x)$ are negative. The case $k = 0$ is Tverberg's classical theorem.

3:30-4:15

Florian Frick (Cornell University)

Geometry and combinatorics of fair division

Abstract. Sperner's lemma is a combinatorial version of Brouwer's fixed point theorem and states that if the vertices of a triangulation of an n-simplex are colored by $n+1$ colors following certain simple rules, then there is a face of the triangulation that exhibits all $n+1$ colors on its vertices. This talk focuses on extensions of Sperner's lemma and related results and applications to problems of fair division. In particular, this approach yields an algorithm for how to fairly divide rent such that n frugal roommates with subjective preferences will not be envious of each other even if the preferences of one roommate are unknown.

This is joint work with Megumi Asada, Kelsey Houston-Edwards, Frédéric Meunier, Vivek Pisharody, Maxwell Polevy, David Stoner, Ling Hei Tsang, and Zoe Wellner.

Wednesday (May 3)
Afternoon session

4:30-5:15

Pablo Soberón (Northeastern University)

Robust Tverberg results via the probabilistic method

Abstract. Tverberg's theorem gives the minimum number $T = T(k, d)$ such that for any set of T points in $\mathbb{R}^d$ we can guarantee the existence of a partition into k sets such that the convex hulls of the parts intersect. During this talk we will be interested in a version with tolerance of Tverberg's theorem. Namely, we wish to bound the minimum number of points $T = T(k, d, t)$ such that for any set of T points in $\mathbb{R}^d$, there is a partition of them into k sets such that even if we remove any t points, the convex hulls of what is left in each part intersect.

We will show how we can apply the probabilistic method to improve the asymptotic bounds on $T(k, d, t)$, as well as how this technique applies the colorful version of Tverberg's theorem and for "colorful Carathéodory" type results.

Thursday (May 4)
Morning session

9:30-10:15

Alexander Barvinok (University of Michigan)

The symmetric moment curve

Abstract. I plan to discuss various interesting and, to a large extent, mysterious properties of the symmetric moment curve

$$t \longmapsto (\cos t,\ \sin t,\ \cos 3t,\ \sin 3t,\ \ldots, \cos(2k-1)t,\ \sin(2k-1)t)$$

in $\mathbb{R}^{2k}$. For $k = 2$, the convex hull of the curve was described by Smilanski, for general k it was used in a joint work with Isabella Novik and Seung Jin Lee to construct $2k$-dimensional centrally symmetric polytopes with many faces, though long before that it appeared in Nudelman's work as a solution to an isoperimetric problem.

Thursday (May 4)
Morning session

10:30-11:15

Andrey Vesnin (Sobolev Institute of Mathematics, Novosibirsk, Russia)

Pogorelov polyhedra from combinatorial, geometrical and topological points of views

Abstract. Let $\mathcal{R}$ be the class of combinatorial 3-dimensional polytopes of simple combinatorial type, different from a tetrahedron, without 3- and 4-belts of faces. In particular, $\mathcal{R}$ contains the dodecahedron and fullerenes. By the results by Pogorelov (1967) and Andreev (1970), $\mathcal{R}$ is exactly the class of polytopes which can be realised in a hyperbolic 3-space with all dihedral angles equal to $\pi/2$. Polyhedra from $\mathcal{R}$ can be used as nice building blocks to construct closed hyperbolic 3-manifolds. The first example of a closed orientable hyperbolic 3-manifold was constructed by Löbell (1931) from eight copies of a 14-hedron from $\mathcal{R}$.

We will discuss the combinatorial and geometrical properties of polyhedra from $\mathcal{R}$ and will present results about their hyperbolic volumes. Also, we will talk on topological properties of hyperbolic 3-manifolds obtained by 4-colourings of that polyhedra.

Thursday (May 4)
Morning session

11:30-12:15

András Bezdek (Auburn University)

On W. Kuperberg's 6 cylinder problem

Abstract. Various problems on infinite long circular cylinders will be surveyed. The talk will focus on a question of W.Kuperberg, where he conjectured that at most 6 unit-radius cylinders with mutually disjoint interiors can touch a unit ball. Heppes and Szabó proved in 1991 that he maximum number is at most 8; nine years later Brass and Wenk improved this result to 7. In this talk an outline of a non-computational solution will be given.

Thursday (May 4)
Afternoon session

2:30-3:15

William J. Martin (Worcester Polytechnic Institute)

Almost orthogonal vectors

Abstract. How many lines may one arrange through the origin in$\mathbb{R}^{10}$ in such a way that any two of them are at least $89°$ apart? How about in $\mathbb{R}^{57}$? Or $\mathbb{R}^n$ as n tends to infinity?

Using this simply stated problem as a point of departure, I will explore various areas of pure and applied mathematics where "almost orthogonal" vectors play a role. Our discussion will touch on some challenging unsolved problems and some important questions. We will have the opportunity to discuss problems in combinatorics, geometry, algebra, communications, quantum information theory, and compressive sensing (which might be viewed as a subdiscipline of data science). While some fairly simple jargon will be introduced, the talk should be accessible to a general mathematics audience.

Thursday (May 4)
Afternoon session

3:30-4:15

Hirotake Kurihara (National Institute of Technology, Kitakyushu College)

On graph structures on great antipodal sets of Hermitian symmetric spaces

Abstract. For an irreducible Hermitian symmetric space of compact type, there is a "good" finite subset of the space, which is called a great antipodal set. In this talk, we will explain the following fact: when we make a graph from each great antipodal set in a "natural way", the graph must be distance-transitive. Also we will give some remarks for relations between data of the graph and data of the ambient space. This is a joint work with Takayuki Okuda (Hiroshima University).

Thursday (May 4)
Afternoon session

4:30-5:15

Wei-Hsuan Yu (Michigan State University)

New bounds for equiangular lines and spherical two-distance sets

Abstract. The set of points in a metric space is called an s-distance set if pairwise distances between these points admit only s distinct values. Two-distance spherical sets with the set of scalar products $\{\alpha, -\alpha\}$, $\alpha \in [0, 1)$, are called equiangular. The problem of determining the maximal size of s-distance sets in various spaces has a long history in mathematics. We determine a new method of bounding the size of an s-distance set in two-point homogeneous spaces via zonal spherical functions. This method allows us to prove that the maximum size of a spherical two-distance set in $\mathbb{R}^n$ is $\frac{n(n+1)}{2}$ with possible exceptions for some $n = (2k+1)^2 - 3$, $k \in \mathbb{N}$. We also prove the universal upper bound $\sim \frac{2}{3}na^2$ for equiangular sets with $\alpha = \frac{1}{a}$ and, employing this bound, prove a new upper bound on the size of equiangular sets in an arbitrary dimension. Finally, we classify all equiangular sets reaching this new bound.

Friday (May 5)
Morning session

9:30-10:15

Rupei Xu (University of Texas at Dallas)

An Interactive Proof Study of Erdős-Szekeres Conjecture

Abstract. In 1935, Erdős and Szekeres proved that for every integer $n \geq 3$, there is a minimal integer $ES(n)$ such that any set of $ES(n)$ points in the plane in general position contains n points in convex position. They showed that $ES(n) \geq 2^{n-2}+1$ and conjectured this to be sharp.

There are many different variations of Erdős-Szekeres Conjecture. Two player game variant was studied by Kolipaka and Govindarajan in 2012.

In this paper, for the first time, the *Interactive Proof* Method was introduced to investigate the Erdős-Szekeres Conjecture. Compared to the two player game variant, interactive proof method uses more probabilistic properties of the distribution of points. The connection of this method to the design of randomized and deterministic algorithms to find the convex hull of n points for a given point set is also discussed.

Friday (May 5)
Morning session

10:30-11:15

Brian Kodalen (Worcester Polytechnic Institute)

Connectivity of Nearest Neighbor Graphs

Abstract. Symmetric association schemes can be viewed as edge-colorings of a complete graph with many regular properties. Denoting one such color as the “nearest relation", we are able to form a graph induced by that color. One question that arises in this context is the connectivity of such a graph. Godsil conjectured that the smallest cut-set has cardinality equal to the valency of the graph however only half the valency has been shown thus far. In this talk we give a brief introduction to association schemes and their regularity properties. We will then use these properties to examine a specific cut-set given by the neighborhood of any vertex. We will show that deleting this neighborhood with the original vertex leaves behind at most one non-singleton component with isolated vertices only arising in specific cases. We then use this result to show that a vertex and any proper subset of its neighborhood can never serve as a cut-set for our graph as well as examine the implications of such a result.

Friday (May 5)
Morning session

11:30-12:15

Alexey Balitskiy (Massachusetts Institute of Technology & Moscow Institute of Physics and Technology)

Elementary billiard technique applied in convex and symplectic geometry

Abstract. One can consider a dynamical system: a smooth convex billiard table $K \subset V = \mathbb{R}^n$, where the reflection law may be classical (if V is Euclidean) or generalized (if V is an arbitrary normed space with the unit ball $T^\circ \subset V$). The length $\xi_T(K)$ of the shortest closed billiard trajectory on such a table gives a peculiar characteristic of the pair (K, T) of convex bodies. In many cases, this value may be computed using a beautiful elementary geometric argument by D. Bezdek and K. Bezdek. In the talk I'm going to present this billiard technique and show some of its consequences.

One etude is about the existence of closed billiard trajectories in simplices: the approach of Bezdek&Bezdek gives a short proof that in a simplex whose dihedral angles are acute there exists a closed billiard trajectory (with bounce points only in the relative interiors of the facets). Surprisingly, it is still unknown if there is a closed billiard trajectory in any obtuse-angled triangle.

Another etude is motivated by the symplectic approach (developed by S. Artstein-Avidan, Y. Ostrover, and R. Karasev) to famous Mahler's conjecture. They established that $\xi_T(K)$ is related to the so-called symplectic capacity of $K \times T$. After that they reduced Mahler's conjecture to Viterbo's conjecture from symplectic geometry, which can be reformulated in a particular case as follows: For any convex bodies $K, T \subset \mathbb{R}^n$,

$$\mathrm{vol}(K \times T) \geq \frac{\xi_T(K)^n}{n!}.$$

Time permitting, I will discuss some interesting examples of equality in Viterbo's conjectured inequality, including the case of K permutohedron.