### Schedule of talks.

Wednesday (May 4) Morning session

9:30-10:15

# Charles Radin (The University of Texas at Austin) Rigidity of many-component systems

Abstract. We consider systems made of many interacting components, both packings of many bodies, and networks of many edges. There are traditional optimization programs for both subjects: densest packings for the former and extremal graphs for the latter. We discuss an outgrowth of these optimization programs: the emergence of phases as the size of the system grows, and the way global quantities such as rigidity are analyzed for large systems.

#### Wednesday (May 4) Morning session

#### 10:25-11:10

# Art Duval (The University of Texas at El Paso) A non-partitionable Cohen-Macaulay simplicial complex

**Abstract.** A long-standing conjecture of Stanley states that every Cohen-Macaulay simplicial complex is partitionable. We disprove the conjecture by constructing an explicit counterexample in three dimensions. Due to a result of Herzog, Jahan and Yassemi, our construction also disproves the conjecture that the Stanley depth of a monomial ideal is always at least its depth.

Joint work with Bennet Goeckner, Carly Klivans, and Jeremy Martin.

#### Wednesday (May 4) Morning session

#### 11:20-12:05

# Timo de Wolff (Texas A&M University)

### New Certificates for Nonnegativity via Circuit Polynomials and Geometric Programming

**Abstract.** Deciding nonnegativity of real polynomials is a key question in real algebraic geometry with crucial importance in polynomial optimization. Since this problem is NP-hard, one is interested in finding sufficient conditions (certificates) for nonnegativity, which are easier to check. Since the 19th century the standard certificates are sums of squares (SOS); see particularly Hilbert's 17th problem.

In this talk, we introduce *polynomials supported on circuits*. For this class nonnegativity is characterized by an invariant, which can be derived from the initial polynomial immediately. In consequence, we obtain an *entirely new class* of nonnegativity certificates, which are *independent* of SOS certificates.

In practice, one uses semidefinite programming (SDP), which is based on SOS certificates, as the standard method to solve polynomial optimization problems. Similar as SOS correspond to SDP our certificates correspond to geometric programming (GP). We show that our certificates yield GPs which efficiently compute lower bounds both for unconstrained and constrained polynomial optimization problems. Particularly, our approach is significantly faster and often provides better bounds than semidefinite programming,

The talk is based on joint work with Sadik Iliman and Mareike Dressler.

2:00-2:45

#### Włodzimierz Kuperberg (Auburn University)

Double-lattice packings revisited – recent developments

Abstract. A parallelogram inscribed in a given convex disk K in the plane is said to be *extensive* if each of its sides is at least as long as one-half of the affine diameter of K parallel to the side. The notion of extensive parallelograms was introduced in [1] (1990) to produce dense double-lattice packings of the plane with an arbitrary convex disk K. In particular, it was proven there that each such K admits a double-lattice packing of density at least  $\sqrt{3}/2 = 0.866...$  Also, the densest double-lattice packing was presented there for the regular pentagon and the regular heptagon, of density  $(5 - \sqrt{5})/3 = 0.92131...$ and 0.8926..., respectively. We conjectured that the double-lattice packing of the regular pentagon is the densest one among all packings with its congruent copies, not just of the double-lattice kind. The conjecture still remains open. Very recent, partial results towards a proof, by Kushner, Kallus, and Hales will be described, and some new questions will be asked in relation to the old notions and results.

Reference:

[1] G. Kuperberg and W. Kuperberg. *Double-lattice packings of convex bodies in the plane*. Discrete & Computational Geometry, 5(1):389–397, 1990.

2:55-3:40

## Anton Dochtermann (The University of Texas at Austin) Warmth and edge spaces of graphs

Abstract. In recent years two novel approaches for finding lower bounds on the chromatic number of a graph have been introduced. One involves studying the topological connectivity of the 'edge space' of a graph, dating back to Lovasz's celebrated proof of the Kneser conjecture. The other is motivated by constructions in statistical physics and involves the notion of long range action of random branching walks and the 'warmth' of a graph, as introduced by Brightwell and Winkler.

We seek to relate these two constructions, and in particular we provide evidence for the conjecture that the warmth of a graph G is always less than three plus the connectivity of its edge space. We succeed in establishing the first nontrivial case of the conjecture, and calculate the warmth of a family of graphs with relevant edge space topology. We also demonstrate how the local structure of a graph involving bipartite subgraphs influences warmth, providing an analogue for a similar result in the context of edge spaces. This is joint work with Ragnar Freij.

4:00-4:45

## Christopher O'Neill (Texas A&M University) Shifting numerical monoids

**Abstract.** Consider the family of numerical monoids  $S_n = \langle n, n + r_1, \ldots, n + r_k \rangle \subset \mathbb{N}$  obtained by varying n. In this talk, we exhibit periodic behavior of the minimal presentations of  $S_n$  when n is sufficiently large. As a consequence, we characterize the eventual behavior of several arithmetic quantities arising in factorization theory.

4:55-5:40

## Wei-Hsuan Yu (Michigan State University) Recent progress on equiangular lines

Abstract. A set of lines in  $\mathbb{R}^n$  is called equiangular if the angle between each pair of lines is the same. We address the question to determine the maximum size of equiangular lines in  $\mathbb{R}^n$ . I will talk about the recent progress on the upper bounds of equiangular line problems. In particular, we proved that there are no 76 equiangular lines in  $\mathbb{R}^{19}$  and starting from the dimension  $n = (2k + 1)^2 - 2$ , for any positive integer k, there will be long range fixed number of upper bounds for equiangular lines.

#### Thursday (May 5) Morning session

9:30-10:15

# **Vladimir Vershinin** (University of Montpellier) Braids and homotopy groups of spheres

Abstract. We start with the geometrical (naive) definition of braids and then identify them with the fundamental group of configuration space of a manifold. The case of a surface is particular interesting. We recall some classical properties of braids and then pass to Brunnian braids. A braid is Brunnian if it becomes trivial after removing any one of its strands. We describe algebraically the group of Brunnian braids on a general surface, if the surface is not a sphere or projective plane. In these exceptional cases the group of Brunnian braids is described by an algebraic procedure together with the homotopy groups of a 2-sphere. If there will be time we shall speak about the graded Lie algebra of the descending central series related to Brunnian braid group. It is proved that this is a free Lie algebra and the set of free generators is described.

#### Thursday (May 5) Morning session

#### 10:25-11:10

### **Károly Bezdek** (University of Calgary) On non-separable families of positive homothetic convex bodies

Abstract. A finite family  $\mathcal{B}$  of balls with respect to an arbitrary norm in  $\mathbb{R}^d$   $(d \ge 2)$  is called a non-separable family if there is no hyperplane disjoint from  $\bigcup \mathcal{B}$  that strictly separates some elements of  $\mathcal{B}$  from all the other elements of  $\mathcal{B}$  in  $\mathbb{R}^d$ . In this talk we prove that if  $\mathcal{B}$  is a non-separable family of balls of radii  $r_1, r_2, \ldots, r_n$   $(n \ge 2)$ with respect to an arbitrary norm in  $\mathbb{R}^d$   $(d \ge 2)$ , then  $\bigcup \mathcal{B}$  can be covered by a ball of radius  $\sum_{i=1}^n r_i$ . This was conjectured by Erdős for the Euclidean norm and was proved for that case by A. W. Goodman and R. E. Goodman [Amer. Math. Monthly 52 (1945), 494-498]. On the other hand, in the same paper A. W. Goodman and R. E. Goodman conjectured that their theorem extends to arbitrary nonseparable finite families of positive homothetic convex bodies in  $\mathbb{R}^d$ ,  $d \ge 2$ . Besides giving a counterexample to their conjecture, we prove that conjecture under various additional conditions. This is a joint work with Zs. Lángi (Univ. of Tech., Budapest). Thursday (May 5) Morning session

11:20-12:05

### Brandt Kronholm (The University of Texas Rio Grande Valley) A Supercrank for P(n, 3) modulo Primes of the form 6j - 1

Abstract. In 1944, Dyson called for *direct* proofs of Ramanujan's congruences for p(n) that give concrete demonstrations of how the associated partitions can be systematically divided into equinumerous classes. He conjectured that a very simple statistic on partitions, called the "rank" of a partition, performs this division when considered modulo 5 and 7. In the same paper, Dyson hypothesized the existence of a different statistic, called the "crank," that would witness Ramanujan's congruence modulo 11 in the same way. Dyson's conjectures on ranks and cranks have since been proven.

Recent results show that Dyson's ideas can be applied successfully to partitions of n into exactly d parts, denoted by P(n, d). Moreover, some of these new cranks for P(n, d) have a very surprising quality that is not shared with those for p(n); there are cranks for P(n, d)that witness each and every instance of divisibility modulo a given prime. We call these cranks supercranks.

In this talk, we make use of Ehrhart Geometry and other techniques to prove the following result:

**Theorem.** Largest part minus smallest part is a supercrank for  $P(n,3) \pmod{m}$  where m is any prime of the form 6j - 1.

This talk is joint work with Felix Breuer and Dennis Eichhorn.

2:00-2:45

# Lorenzo Sadun (The University of Texas at Austin) Bipodal random graphs above Erdos-Renyi

Abstract. We consider large dense random graphs on N vertices in which the density of an associated sub-graph H (say, a triangle) is slightly higher than would be expected for an Erdos-Renyi graph. We prove that as  $N \to \infty$ , the structure of the random graph becomes "bipodal". This means that there are two classes of vertices, say red and blue, and fixed edge probabilities for red-red, red-blue, and blueblue edges. Furthermore, this phenomenon is universal, in that similar results apply for all choices of the sub-graph H.

2:55-3:40

### **Timothy Huber** (The University of Texas Rio Grande Valley) McKay-Thompson series and high level approximations for $\pi$

**Abstract.** We systematize the construction of Ramanujan-Sato series involving expansions for modular forms in terms of McKay-Thompson series. This work extends lower level results, with level 17 as a prototype. We demonstrate an explicit construction of the differential equations in the general case; discuss how to optimize the recursion defining the series coefficients; and relate an algorithm for finding all singular values of the Hauptmoduln within the radius of convergence via the construction of explicit modular equations.

#### 4:00-4:45

### Jacob White (The University of Texas Rio Grande Valley) Ehrhart-Hilbert polynomials arising from combinatorial Hopf monoids

**Abstract.** The chromatic polynomial of a graph, and the order polynomial of a poset are wonderful invariants. They arise in the study of combinatorial Hopf algebras, Ehrhart theory, and commutative algebra. We will review work of Aguiar, Bergeron, and Sottile, who showed that every combinatorial Hopf algebra comes equipped with a 'nice' homomorphism to the polynomial ring, generalizing these examples. I will discuss how, under certain conditions, these same polynomials arise as Ehrhart-Hilbert polynomials of certain simplicial complexes, and discuss ongoing work regarding the combinatorics of these complexes.

4:55-5:40

## Pablo Soberón (Northeastern University) Quantitative and colorful combinatorial geometry

**Abstract.** We review some classic results in combinatorial geometry from a quantitative perspective. In other words, we look for variants of Helly's, Caratheodory's and Tverberg's theorem guaranteeing the that the size of the sets involved and their intersections are large. We will discuss what proof methods work for the colorful extensions of some of these results, and how they change depending on what function we use to measure our convex sets.

#### Friday (May 6) Morning session

9:30-10:15

# **Elena Poletaeva** (The University of Texas Rio Grande Valley)

On pyramids, Yangians and finite W-algebras

**Abstract.** A finite W-algebra is certain associative algebra attached to a pair  $(\mathfrak{g}, e)$ , where  $\mathfrak{g}$  is a complex semisimple Lie algebra and  $e \in \mathfrak{g}$  is a nilpotent element. It is a generalization of the universal enveloping algebra  $U(\mathfrak{g})$ .

Certain combinatorial objects called pyramids encode the information needed to define finite W-algebras for  $\mathfrak{gl}(n)$ . These algebras were described in terms of Yangians (a class of Hopf algebras) by J. Brundan and A. Kleshchev. Then J. Brown, J. Brundan and S. Goodwin generalized this approach to the Lie superalgebras  $\mathfrak{gl}(m|n)$ .

We study finite W-algebras for the queer Lie superalgebra Q(n)and discuss the connection between finite W-algebras and Yangians.

It is a joint work with V. Serganova.

Friday (May 6) Morning session

10:25-11:10

### Matthew Haulmark (University of Wisconsin-Milwaukee) Boundaries of CAT(0) Groups with Isolated Flats and The Kapovich-Kleiner Theorem

Abstract. In 2000 Kapovich and Kleiner proved that if G is a one-ended hyperbolic group that does not split over a two-ended subgroup, then the boundary of G is either a Menger curve, a Sierpinski carpet, or a circle. In this talk I will discuss CAT(0) spaces and their boundaries, as well as what it means for a group to be CAT(0). I will also provide a generalization of the Kapovich and Kleiner theorem to the isolated flats setting. This work is the topic of my dissertation under the advisement of G. Christopher Hruska.