Schedule of talks.

Monday (September 23)
Morning session

9:00-9:45

Jesús A. De Loera (University of California, Davis)

Tverberg-type theorems with altered nerves and problems of Data classification

Abstract. The classical Tverberg's theorem says that a set with sufficiently many points in R^d can always be partitioned into m parts so that the (m - 1)-simplex is the (nerve) intersection pattern of the convex hulls of the parts. Our main results demonstrate that Tverberg's theorem is but a special case of a much more general situation. Given sufficiently many points, any tree or cycle, can also be induced by at least one partition of the point set. The proofs require a deep investigation of oriented matroids and order types. Our results are strongly related to the classification of data.

Joint work with Deborah Oliveros, Tommy Hogan, Dominic Yang (supported by NSF).

Monday (September 23)
Morning session

10:00-10:45

Catherine Yan (Texas A&M University)

Vector Parking Functions with Periodic Boundaries and Rational Parking Functions

Abstract. Vector parking functions are sequences of non-negative integers whose order statistics are bounded by a given integer sequence $\mathbf{u} = (u_0, u_1, u_2, \dots)$. Using the theory of fractional power series and an analog of Newton-Puiseux Theorem, we derive the exponential generating function for the number of $\mathbf{u}$-parking functions when $\mathbf{u}$ is periodic. Our method is to convert an Appell relation of Gončarov polynomials to a system of linear equations. Solving the system we obtain an explicit formula of the exponential generating function in terms of Schur functions of certain fractional power series. In particular, we apply our methods to rational parking functions for which the boundary is induced by a linear function with rational slope.

Monday (September 23)
Morning session

11:00-11:45

Hiroshi Nozaki (Aichi University of Education)

Maximal 2-distance sets containing the regular simplex

Abstract. A finite subset X of the Euclidean space is called an s-distance set if the number of the distances of two distinct vectors in X is equal to s. An s-distance set X is said to be maximal if any vector cannot be added to X while maintaining s-distance. We investigate a necessary and sufficient condition for vectors to be added to the regular simplex such that the set has only 2 distances. We construct several maximal 2-distance sets that contain the regular simplex. In particular, there exist infinitely many maximal non-spherical 2-distance sets that contain both the regular simplex and the representation of a strongly resolvable design. The maximal 2-distance set has size $2k^2(k+1)$, and the dimension is $d=(k-1)(k+1)^2-1$, where k is a prime power.

Monday (September 23)
Afternoon session

2:00-2:45

Pavel Galashin (University of California, Los Angeles)

Higher Secondary Polytopes and Regular Plabic Graphs

Abstract. Given a configuration A of n points in $\mathbb{R}^{d-1}$, we introduce the *higher secondary polytopes* $\Sigma_{A,1}, \ldots, \Sigma_{A,n-d}$, which have the property that $\Sigma_{A,1}$ agrees with the secondary polytope of Gelfand–Kapranov–Zelevinsky, while the Minkowski sum of these polytopes agrees with Billera–Sturmfels' fiber zonotope associated with (a lift of) A. In a special case when $d = 3$, we refer to our polytopes as *higher associahedra*. They turn out to be related to the theory of total positivity, specifically, to certain combinatorial objects called *plabic graphs*, introduced by Postnikov in his study of the totally positive Grassmannian. We define a subclass of *regular* plabic graphs and show that they correspond to the vertices of the higher associahedron $\Sigma_{A,k}$, while *square moves* connecting them correspond to the edges of $\Sigma_{A,k}$. Finally we connect our polytopes to *soliton graphs*, the contour plots of soliton solutions to the KP equation, which were recently studied by Kodama and Williams. In particular, we confirm their conjecture that when the higher times evolve, soliton graphs change according to the moves for plabic graphs. Joint work with Alexander Postnikov and Lauren Williams.

3:00-3:45

Anton Dochtermann (Texas State University)

Exposed circuits, chordal clutters, and extendably shellable complexes

Abstract. Chordal graphs are widely studied combinatorial objects with many applications and characterizations. They also make an appearance in commutative algebra in the context of Fröberg's theorem, which says that a graph is chordal if and only a certain underlying quadratic ideal has a 'linear resolution'. A number of authors have provided higher-dimensional analogues of this result, describing combinatorial classes of d-clutters whose associated circuit ideals satisfy certain homological properties. Inspired by recent results in chordal graphs we show that 'linear quotients' of such ideals can be characterized in terms of removing 'exposed circuits' in the complement clutter. This leads to a notion of higher dimensional 'chordal complexes' that borrows from simple homotopy theory and commutative algebra. We investigate applications of our results including a connection to Simon's conjecture, which posits that the k-skeleta of a simplex are extendably shellable. For example in joint work with Jared Culbertson, Dan Guralnik, and Peter Stiller we show that any shellable d-dimensional simplicial complex with $d + 3$ vertices is extendably shellable.

Monday (September 23)
Afternoon session

4:00-4:45

Acadia Larsen (University of Texas Rio Grande Valley)

Combinatorial Proofs for Partition Identities and Divisibility Properties for Partitions of n with at Most m Parts

Abstract. We show for a prime power number of parts m that the first differences of partitions into at most m parts can be expressed as a non-negative linear combination of partitions into at most $m-1$ parts. To show this relationship, we combine a quasipolynomial construction of $p(n, m)$ with a new partition identity for a finite number of parts. Furthermore, we use this combinatorial interpretation to provide "universal" bijective proofs for divisibility properties of $p(n, m)$.

Tuesday (September 24)
Morning session

9:00-9:45

Lorenzo Sadun (University of Texas at Austin)

Nucleation in Random Graphs

Abstract. Ensembles of large random graphs with hard constraints on the numbers of edges and triangles exhibit a number of very different phases. At a previous South Texas Discrete Geometry conference, I reported on the overall phase portrait and on what a typical graph in each of these phases looks like. After reviewing those results, I will report on recent progress on the way that a large but finite graph can change form from one phase to another, a process (vaguely) analogous to the nucleation of crystals as a fluid cools. This is joint work with Charles Radin and Joe Neeman.

10:00-10:45

Nathan Williams (University of Texas at Dallas)

Fixed Points of Parking Functions

Abstract. We define an action of words in $[m]^n$ on $\mathbb{R}^m$ to give a new characterization of rational parking functions—they are exactly those words whose action has a fixed point. We use this viewpoint to give a simple definition of Gorsky, Mazin, and Vazirani's zeta map on rational parking functions when m and n are coprime, and prove that this zeta map is invertible. A specialization recovers Loehr and Warrington's sweep map on rational Dyck paths. This is joint work with Jon McCammond and Hugh Thomas.

Tuesday (September 24)
Morning session

11:00-11:45

Masanori Sawa (Kobe University)

On a certain system of Diophantine equations and Gaussian designs

Abstract. In this talk we first introduce a certain system of Diophantine equations, originally designed as Hausdorff's simplification of Hilbert's solution of Waring problem, and then describe a close connection with quasi-Hermite polynomials in special function theory, Gaussian designs in algebraic combinatorics and Christoffel-Darboux kernels in functional analysis. We show some nonsolvability theorems for our equations. This is a joint work with Yukihiro Uchida (Tokyo Metropolitan University).

Tuesday (September 24)
Afternoon session

2:00-2:45

Dmitriy Bilyk (University of Minnesota)

Discrete minimizers of energies on the sphere

Abstract. The phenomenon of clustering of optimal measures for various interaction energies has been observed both numerically and theoretically: for certain types of interactions $f(x, y)$ (for example, for *attractive-repulsive* potentials) the minimum of the continuous energy integral

$$\int_\Omega \int_\Omega f(x, y) d\mu(x) d\mu(y)$$

over all Borel probability measures is attained by discrete measures. We shall discuss various manifestations of this phenomenon, with a particular emphasis on the case when $\Omega = \mathbb{S}^{d-1}$ is a sphere. One of the most interesting examples in this case is the *p-frame energy*, corresponding to the potential $f(x, y) = |\langle x, y \rangle|^p$, $p > 0$. When certain highly symmetric configurations (*tight designs*) exist, they provide discrete minimizers of this energy for certain ranges of p, and it is conjectured that all minimizers are discrete whenever $p \notin 2\mathbb{N}$. Other examples of this type include the Fejes Tóth conjecture on the sum of line angles, the causal variational principle from mathematical physics, and other energies.

Tuesday (September 24)
Afternoon session

3:00-3:45

Timothy Huber (University of Texas Rio Grande Valley)

Partition Congruences Associated with Modular Forms of Level 7

Abstract. Congruences and other identities are derived for a set of colored partition functions associated with modular forms of level 7. The congruences result from linear transformations between vector spaces of modular forms of weight 1. Basis elements for the vector spaces are derived from a computer search for theta quotients arising from elliptic functions of a specified form.

Wednesday (September 25)
Morning session

9:00-9:45

Art Duval (University of Texas at El Paso)

Counting topologies of metric of holomorphic polynomial field with simple zeros

Abstract. We reduce the problem in the title to the problem of counting unlabeled planar trees with black and white vertices, where each white vertex has degree at least three, and where no white vertices are adjacent. We use the theory of species to describe a generating function in terms of a root of a cubic equation; this is good enough for a computer algebra system to explicitly compute the number of trees with m black vertices and n white vertices for many small values of m and n. In particular, we give the number of trees up to $m = 13$ black vertices. A key tool is an extension of the Dissymmetry Theorem to certain multi-sort species.

This is joint work with Martín Eduardo Armenta-Frías.

Wednesday (September 25)
Morning session

10:00-10:45

Greta Panova (University of Southern California)

Hook formulas for skew shapes

Abstract. In 2014, Naruse announced a formula for skew shapes as a positive sum of products of hook-lengths using "excited diagrams" coming from the Algebraic Geometry of the Grassmannian. We will show several combinatorial and algebraic proofs of this formula. Multivariate versions of the hook formula lead to exact product formulas for certain skew SYTs and evaluations of Schubert polynomials. The Naruse hook-length formula can also be used to derive asymptotic results for the $f^{\lambda/\mu}$'s in many regimes, and principal evaluations of certain Schubert polynomials shedding light over Stanley's “Schubert shenanigans” conjectures. Joint work with Alejandro Morales and Igor Pak.

Wednesday (September 25)
Morning session

11:00-11:45

Zhenyang Zhang (University of California, Davis)

On the Diameters of Oriented Matroids

Abstract. Motivated by the famous open question of the complexity of the *simplex method* and of the *criss-cross method*, we investigate the diameter of the cocircuit graph of an oriented matroid. An important special case is that of *realizable* oriented matroids where this project just considers the graph of the hyperplane arrangement.

The *diameter* of $\mathcal{M}$ is defined as the largest distance between two vertices on $G_{\mathcal{M}}$. We denote by $\Delta(n,r)$ the length of the largest diameter on the graphs corresponding to $G_{\mathcal{M}}$ over all oriented matroid $\mathcal{M} = (E, \mathcal{C}^*)$ with $|E| = n$ and rank r. I will present a quadratic bound for all oriented matroids, and some other improved bounds for special families of oriented matroids. Joint work with Ilan Adler (Berkeley), Jesús A. De Loera (Davis), and Steve Klee (Seattle).

Wednesday (September 25)
Afternoon session

2:00-2:45

Nikolay Dolbilin (Steklov Mathematical Institute and Moscow State University)

$2R$-isometrical Delone Sets

Abstract. In a Delone set $X \subset \mathbf{R}^d$ with parameters r and R, the R can be interpreted as the radius of the largest ball free of points from X and the $2r$ as the shortest interpoint distance in X. A Delone set X is a *regular system* if its symmetry group $\mathrm{Sym}(X)$ is point-transitive. A subset $C_x(\rho) := \{y \in X : |xy| \leq \rho\}$ is called a *ρ-cluster*. Two ρ-clusters $C_x(\rho)$ and $C_{x'}(\rho)$ are *equivalent* if there is an isometry g of $\mathbb{R}^d$ such that $g(x) = x'$ and $g(C_x(2R)) = C_{x'}(2R)$. It is obvious that in a regular system X for each $\rho > 0$ all ρ-clusters are pairwise equivalent. Let $S_x(2R)$ denote the symmetry group of the $2R$-cluster.

Which radius $\hat{\rho}_d$ should be taken so that the mutual equivalence of $\hat{\rho}_d$-clusters $C_x(\hat{\rho}_d)$ for all $x \in X$ would guarantee the regularity of the Delone set X? – is one of central problems of the local theory for regular systems.

A Delone set X is *$2R$-isometrical* if $2R$-clusters $C_x(2R)$ for all $x \in X$ are pairwise equivalent. In a $2R$-isometrical set the groups $S_x(2R)$ of $2R$-clusters for $x \in X$ are conjugate to each other in the group $\mathrm{Iso}(d)$. For any $d \geq 2$, a $2R$-isometrical set $X \subset \mathbb{R}^d$ is not necessarily a regular system. Nevertheless, the study of $2R$-isometrical sets and of the groups $S_x(2R)$ is a very important task in the context of regular systems and obtaining new estimates for the regularity radius. In the talk we will discuss results on $2R$-isometrical sets and regular systems obtained within the local theory of regular systems.

Wednesday (September 25)
Afternoon session

3:00-3:45

Josiah Park (Georgia Tech)

Designs, Lattices, and Energy Optimization

Abstract. Lattices and finite vector systems hold important roles in various packing questions, which in turn have implications in many scientific disciplines. A natural question concerning both objects is what properties lattices inherit from the vectors that generate them? Or in other words, what restrictions must hold for lattices generated from structured systems? Desirable properties of such lattices might be that the shortest vectors are "well-spread" (and that these vectors also act as generators). Another well-motivated question, dropping consideration of lattices, asks how to spread lines (through the origin) or points on a sphere so as to minimize "energy". The behavior of minimizers for interaction energies of the form

$$I_p(\mu) = \int_{\mathbb{S}_{\mathbb{F}}^{d-1}} \int_{\mathbb{S}_{\mathbb{F}}^{d-1}} |\langle x, y \rangle|^p d\mu(x) d\mu(y)$$

provide an interesting case-study. It turns out that when a tight projective t-design exists, equally distributing mass over it gives a minimizer of the quantity I_p on a range between consecutive even integers associated with the strength t. Numerical studies show that several other exceptional objects might be expected to minimize on ranges of p, and that minimizers may be generally discrete when $p > 0$ is not even. I will talk about developments in these areas represented through collaborations with Dmitriy Bilyk, Lenny Fukshansky, Alexey Glazyrin, Ryan Matzke, Deanna Needell, Oleksandr Vlasiuk, and Yuxin Xin.

Thursday (September 26)
Morning session

9:00-9:45

Peter Dragnev (Purdue University Fort Wayne)

Mastodon Theorem - 20 Years in the Making

Abstract. The *Mastodon theorem* (PD., D. Legg, D. Townsend, 2002), establishes that the regular bi-pyramid (North and South poles, and an equilateral triangle on the Equator) is the unique up to rotation five-point configuration on the sphere that maximizes the product of all mutual distances. More generally, given a configuration of points $\{x_1, \ldots, x_N\}$ on the unit sphere in $\mathbb{S}^{n-1} \subseteq \mathbb{R}^n$, its *Riesz s-energy* is defined as

$$\sum_{1\leq i<j\leq N} \frac{1}{\|x_i - x_j\|^s}, \quad s > 0; \quad \sum_{1\leq i<j\leq N} \log \frac{1}{\|x_i - x_j\|}, \quad s = 0.$$

The regular bi-pyramid minimizes the *logarithmic energy* ($s = 0$ case) for five points on $\mathbb{S}^2$.

Optimal point configurations that minimize the s-energy have broad applications in sciences, economics, information theory, etc. Rigorous proofs of optimality are extremely hard though. Even the important Coulomb energy ($s = 1$) case for five points on the unit sphere in 3-D space was resolved only recently (2013) by Richard Schwartz utilizing a computer-aided proof. In a subsequent monograph Schwartz extends the optimality of the bipyramid to all $s < s^*$.

In a joint work with Oleg Musin we generalize the Mastodon Theorem to $n+2$ points on $\mathbb{S}^{n-1}$, namely we characterize all stationary configurations, and show that all local minima occur when a configuration splits in two orthogonal simplexes of k and ℓ vertices, $k + \ell = n + 2$, with global minimum attained when $k = \ell$ or $k = \ell + 1$ depending on the parity of n.

Thursday (September 26)
Morning session

10:00-10:45

Edgardo Roldán-Pensado (Universidad Nacional Autónoma de México)

Equipartitions and the Mahler Conjecture

Abstract. The Mahler volume of a symmetric convex body is the product of its volume and the volume of its dual. There is a conjecture stating the hypercubes have the smallest possible Mahler volume. In 2017, Iriyeh and Shibata published a very long proof in dimension three of this conjecture. I want to discuss a simpler proof in which one of the steps involves solving an interesting equipartition problem. This work is joint with M. Fradelizi, A. Hubard, M. Meyer and A. Zvavitch.

Thursday (September 26)
Morning session

11:00-11:45

Masatake Hirao (Aichi Prefectural University)

Finite frames, frame potentials and determinanal point processes on the sphere

Abstract. In this talk we discuss on frame potentials of several types of determinantal point processes on the sphere, which are used in a fermion model in quantum mechanics and also studied in probability theory. We compare these random point configurations with spherical designs, which are one of the non-random “good" point configurations on the sphere, and also discuss on its applications to numerical integrations. Moreover, we also discuss some other potential energies and discrepancies if possible.

Thursday (September 26)
Afternoon session

2:00-2:45

Jacob White (University of Texas Rio Grande Valley)

Cohen-Macaulay Coloring Complexes

Abstract. A coloring complex is a relative simplicial complex that arise in combinatorics. The first example is the coloring complex of a graph: the chromatic polynomial of a graph is the Hilbert polynomial of the corresponding coloring complex. Hence, information about the coloring complex implies information about chromatic polynomials. However, there are coloring complexes for posets and matroids as well. Our motivation is to find new inequalities for the coefficients of such polynomials. Recently, Sanyal has found new conditions on Hilbert polynomials of relative Cohen-Macaulay complexes.

We are interested in studying minor-closed families of colorings complexes (such as all coloring complexes coming from graphs, posets, or matroids). Our goal is to determine necessary and sufficient *combinatorial* conditions for determining when *all* coloring complexes in a minor-closed family are Cohen-Macaulay. We will report on our partial progress in this area.

Thursday (September 26)
Afternoon session

3:00-3:45

Brandt Kronholm (University of Texas Rio Grande Valley)

Recent Partition Theory Results on the Coefficients of Gaussian Polynomials

Abstract. In this talk we review several results obtained over the past 18 months on the coefficients of Gaussian polynomials with a look towards future research. Gaussian polynomials are also known as the q-binomial coefficients and are often denoted by $\begin{bmatrix} N+m \\ m \end{bmatrix}$. They are the generating functions for partitions of n into at most m parts, no part larger than N, denoted by $p(n, m, N)$.

We prove a general theorem on an infinite family of prime divisibilities for $p(n, m, N)$ and a general result on the largest coefficient of any given Gaussian polynomial. A result on combinatorial statistics related to certain divisibility properties of $p(n, m, N)$ is proved using some polyhedral geometry and integer lattices.