$2R$-isometrical Delone Sets

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Abstract

In a Delone set $X \subset \mathbf{R}^d$ with parameters r and R, the R can be interpreted as the radius of the largest ball free of points from X and the $2r$ as the shortest interpoint distance in X. A Delone set X is a *regular system* if its symmetry group $\mathrm{Sym}(X)$ is point-transitive. A subset $C_x(\rho) := \{y \in X : |xy| \leq \rho\}$ is called a *ρ-cluster*. Two ρ-clusters $C_x(\rho)$ and $C_{x'}(\rho)$ are *equivalent* if there is an isometry g of $\mathbb{R}^d$ such that $g(x) = x'$ and $g(C_x(2R)) = C_{x'}(2R)$. It is obvious that in a regular system X for each $\rho > 0$ all ρ-clusters are pairwise equivalent. Let $S_x(2R)$ denote the symmetry group of the $2R$-cluster.

Which radius $\hat{\rho}_d$ should be taken so that the mutual equivalence of $\hat{\rho}_d$-clusters $C_x(\hat{\rho}_d)$ for all $x \in X$ would guarantee the regularity of the Delone set X? – is one of central problems of the local theory for regular systems.

A Delone set X is *$2R$-isometrical* if $2R$-clusters $C_x(2R)$ for all $x \in X$ are pairwise equivalent. In a $2R$-isometrical set the groups $S_x(2R)$ of $2R$-clusters for $x \in X$ are conjugate to each other in the group $\mathrm{Iso}(d)$. For any $d \geq 2$, a $2R$-isometrical set $X \subset \mathbb{R}^d$ is not necessarily a regular system. Nevertheless, the study of $2R$-isometrical sets and of the groups $S_x(2R)$ is a very important task in the context of regular systems and obtaining new estimates for the regularity radius. In the talk we will discuss results on $2R$-isometrical sets and regular systems obtained within the local theory of regular systems (see [1-9]).

References

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