Exposed circuits, chordal clutters, and extendably shellable complexes

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Abstract

Chordal graphs are widely studied combinatorial objects with many applications and characterizations. They also make an appearance in commutative algebra in the context of Fröberg's theorem, which says that a graph is chordal if and only a certain underlying quadratic ideal has a 'linear resolution'. A number of authors have provided higher-dimensional analogues of this result, describing combinatorial classes of d-clutters whose associated circuit ideals satisfy certain homological properties. Inspired by recent results in chordal graphs we show that 'linear quotients' of such ideals can be characterized in terms of removing 'exposed circuits' in the complement clutter. This leads to a notion of higher dimensional 'chordal complexes' that borrows from simple homotopy theory and commutative algebra. We investigate applications of our results including a connection to Simon's conjecture, which posits that the k-skeleta of a simplex are extendably shellable. For example in joint work with Jared Culbertson, Dan Guralnik, and Peter Stiller we show that any shellable d-dimensional simplicial complex with $d+3$ vertices is extendably shellable.