New bounds for equiangular lines and spherical two-distance sets

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Abstract

The set of points in a metric space is called an s-distance set if pairwise distances between these points admit only s distinct values. Two-distance spherical sets with the set of scalar products $\{\alpha, -\alpha\}$, $\alpha \in [0, 1)$, are called equiangular. The problem of determining the maximal size of s-distance sets in various spaces has a long history in mathematics. We determine a new method of bounding the size of an s-distance set in two-point homogeneous spaces via zonal spherical functions. This method allows us to prove that the maximum size of a spherical two-distance set in $\mathbb{R}^n$ is $\frac{n(n+1)}{2}$ with possible exceptions for some $n = (2k + 1)^2 - 3$, $k \in \mathbb{N}$. We also prove the universal upper bound $\sim \frac{2}{3}na^2$ for equiangular sets with $\alpha = \frac{1}{a}$ and, employing this bound, prove a new upper bound on the size of equiangular sets in an arbitrary dimension. Finally, we classify all equiangular sets reaching this new bound.