Unavoidable crossings in finite coverings

Wlodzimierz Kuperberg
(Auburn University)

Abstract

Two convex disks in the plane are said to *cross each other* if the removal of their intersection causes each disk to fall into disjoint components. The possibility of crossings in (infinite) coverings of the plane with congruent copies of a convex disk presents an obstacle in proving L. Fejes Tóth's conjecture (1950) about the thinnest coverings, analogous to a corresponding theorem for packings. A partially analogous version of the theorem requires the covering to be crossing-free. For about 60 years, it was commonly believed that crossings in a plane covering by congruent convex disks, appear to be counterproductive for producing low density, hence are always avoidable. The first example that illustrated the difficulties in attempts to remove crossings by local, finite rearrangements of the disks was given by A. Heppes (unpublished), followed soon by a more general construction by G. Wegner (1980).

In 2010, we produced an example of a convex pentagon P such that in any minimum density covering of the plane with congruent copies of P crossings must occur. Thus any attempt of proving the conjecture of L. Fejes Tóth must take into account the possibility of unavoidable crossings.

In this talk we consider finite versions of the phenomenon of unavoidable crossings in coverings, in the same vein as the constructions of Heppes and Wegner, and we present a variety of new examples of a similar nature, with short and elementary proofs.

This is a joint work with András Bezdek.