Connectivity of Nearest Neighbor Graphs

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Abstract

Symmetric association schemes can be viewed as edge-colorings of a complete graph with many regular properties. Denoting one such color as the "nearest relation", we are able to form a graph induced by that color. One question that arises in this context is the connectivity of such a graph. Godsil conjectured that the smallest cut-set has cardinality equal to the valency of the graph however only half the valency has been shown thus far. In this talk we give a brief introduction to association schemes and their regularity properties. We will then use these properties to examine a specific cut-set given by the neighborhood of any vertex. We will show that deleting this neighborhood with the original vertex leaves behind at most one non-singleton component with isolated vertices only arising in specific cases. We then use this result to show that a vertex and any proper subset of its neighborhood can never serve as a cut-set for our graph as well as examine the implications of such a result.