Lower bounds for numbers of distinct frequencies of patterns in cut and project sets

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Abstract

In this talk we investigate the problem of understanding, for $r > 1$, the numbers of distinct frequencies of patterns of size r which are possible in d dimensional cut and project sets constructed from $d + 1$ dimensional total spaces. For $d = 1$ this is a problem about Sturmian sequences, and it is a corollary of results of Morse and Hedlund (1930's and 40's), together with the well known Steinhaus theorem (1950's), that the number of distinct frequencies is always at most 3, regardless of what r is. For $d > 1$ the problem depends in part on the shape of the patterns being considered, and it is considerably more difficult. Our main result (joint work with Jens Marklof) is that, for any shape of pattern, and for almost every d dimensional physical space in $\mathbb{R}^{d+1}$, the number of distinct frequencies of patterns of size r is not bounded, as $r \to \infty$. As we will see, this problem is closely related to the Littlewood conjecture in Diophantine approximation. Its solution, which answers a question posed (in a slightly different setting) by Erdős, Geelen, and Simpson, ultimately relies on results from ergodic theory in the space of unimodular lattices in $\mathbb{R}^{d+1}$.