Logarithmic and Riesz Equilibrium for Multiple Sources on the Sphere – the Exceptional Case

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Abstract

In this talk we consider the minimal discrete and continuous energy problems on the unit sphere $\mathbb{S}^d$ in the Euclidean space $\mathbb{R}^{d+1}$ in the presence of an external field due to finitely many localized charge distributions on $\mathbb{S}^d$, where the energy arises from the Riesz potential $1/r^s$ (r is the Euclidean distance) for the critical Riesz parameter $s = d - 2$ if $d \geq 3$ and the logarithmic potential $\log(1/r)$ if $d = 2$. Individually, a localized charge distribution is either a point charge or assumed to be rotationally symmetric. The extremal measure solving the continuous external field problem for weak fields is shown to be the uniform measure on the sphere but restricted to the exterior of spherical caps surrounding the localized charge distributions. The radii are determined by the relative strengths of the generating charges. Furthermore, we show that the minimal energy points solving the related discrete external field problem are confined to this support. For $d-2 \leq s < d$, we show that for point sources on the sphere, the equilibrium measure has support in the complement of the union of specified spherical caps about the sources. Numerical examples are presented to illustrate our results.

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