Tverberg plus minus

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Abstract

We prove a Tverberg type theorem: Given a set $A \subset \mathbb{R}^d$ in general position with $|A| = (r-1)(d+1)+1$ and $k \in \{0,1,\ldots,r-1\}$, there is a partition of A into r sets $A_1,\ldots,A_r$ (where $|A_p| \leq d+1$ for each p) with the following property. The unique $z \in \bigcap_{p=1}^{r} \operatorname{aff} A_p$ can be written as an affine combination of the elements in A_p: $z = \sum_{x \in A_p} \alpha(x)x$ for every p and exactly k of the coefficients $\alpha(x)$ are negative. The case $k=0$ is Tverberg's classical theorem.