Elementary billiard technique applied in convex and symplectic geometry

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Abstract

One can consider a dynamical system: a smooth convex billiard table $K \subset V = \mathbb{R}^n$, where the reflection law may be classical (if V is Euclidean) or generalized (if V is an arbitrary normed space with the unit ball $T^\circ \subset V$). The length $\xi_T(K)$ of the shortest closed billiard trajectory on such a table gives a peculiar characteristic of the pair (K, T) of convex bodies. In many cases, this value may be computed using a beautiful elementary geometric argument by D. Bezdek and K. Bezdek. In the talk I'm going to present this billiard technique and show some of its consequences.

One etude is about the existence of closed billiard trajectories in simplices: the approach of Bezdek&Bezdek gives a short proof that in a simplex whose dihedral angles are acute there exists a closed billiard trajectory (with bounce points only in the relative interiors of the facets). Surprisingly, it is still unknown if there is a closed billiard trajectory in any obtuse-angled triangle.

Another etude is motivated by the symplectic approach (developed by S. Artstein-Avidan, Y. Ostrover, and R. Karasev) to famous Mahler's conjecture. They established that $\xi_T(K)$ is related to the so-called symplectic capacity of $K \times T$. After that they reduced Mahler's conjecture to Viterbo's conjecture from symplectic geometry, which can be reformulated in a particular case as follows: For any convex bodies $K, T \subset \mathbb{R}^n$,

$$\mathrm{vol}(K \times T) \geq \frac{\xi_T(K)^n}{n!}.$$

Time permitting, I will discuss some interesting examples of equality in Viterbo's conjectured inequality, including the case of K permutohedron.