Double-lattice packings revisited – recent developments

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Abstract

A parallelogram inscribed in a given convex disk K in the plane is said to be *extensive* if each of its sides is at least as long as one-half of the affine diameter of K parallel to the side. The notion of extensive parallelograms was introduced in [1] (1990) to produce dense double-lattice packings of the plane with an arbitrary convex disk K. In particular, it was proven there that each such K admits a double-lattice packing of density at least $\sqrt{3}/2 = 0.866\ldots$. Also, the densest double-lattice packing was presented there for the regular pentagon and the regular heptagon, of density $(5 - \sqrt{5})/3 = 0.92131\ldots$ and $0.8926\ldots$, respectively. We conjectured that the double-lattice packing of the regular pentagon is the densest one among all packings with its congruent copies, not just of the double-lattice kind. The conjecture still remains open. Very recent, partial results towards a proof, by Kushner, Kallus, and Hales will be described, and some new questions will be asked in relation to the old notions and results.

Reference:

[1] G. Kuperberg and W. Kuperberg. *Double-lattice packings of convex bodies in the plane.* Discrete & Computational Geometry, 5(1):389–397, 1990.