A Supercrank for $P(n, 3)$ modulo Primes of the form $6j - 1$

Brandt Kronholm
(The University of Texas Rio Grande Valley)

Abstract

In 1944, Dyson called for *direct* proofs of Ramanujan's congruences for $p(n)$ that give concrete demonstrations of how the associated partitions can be systematically divided into equinumerous classes. He conjectured that a very simple statistic on partitions, called the "rank" of a partition, performs this division when considered modulo 5 and 7. In the same paper, Dyson hypothesized the existence of a different statistic, called the "crank," that would witness Ramanujan's congruence modulo 11 in the same way. Dyson's conjectures on ranks and cranks have since been proven.

Recent results show that Dyson's ideas can be applied successfully to partitions of n into exactly d parts, denoted by $P(n, d)$. Moreover, some of these new cranks for $P(n, d)$ have a very surprising quality that is not shared with those for $p(n)$; there are cranks for $P(n, d)$ that witness *each and every* instance of divisibility modulo a given prime. We call these cranks *supercranks.*

In this talk, we make use of Ehrhart Geometry and other techniques to prove the following result:

Theorem. *Largest part minus smallest part is a supercrank for* $P(n, 3)$ $\pmod{m}$ *where* m *is any prime of the form* $6j - 1$.

This talk is joint work with Felix Breuer and Dennis Eichhorn.