New Certificates for Nonnegativity via Circuit Polynomials and Geometric Programming

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Abstract

Deciding nonnegativity of real polynomials is a key question in real algebraic geometry with crucial importance in polynomial optimization. Since this problem is NP-hard, one is interested in finding sufficient conditions (certificates) for nonnegativity, which are easier to check. Since the 19th century the standard certificates are sums of squares (SOS); see particularly Hilbert's 17th problem.

In this talk, we introduce *polynomials supported on circuits*. For this class nonnegativity is characterized by an invariant, which can be derived from the initial polynomial immediately. In consequence, we obtain an *entirely new class* of nonnegativity certificates, which are *independent* of SOS certificates.

In practice, one uses semidefinite programming (SDP), which is based on SOS certificates, as the standard method to solve polynomial optimization problems. Similar as SOS correspond to SDP our certificates correspond to geometric programming (GP). We show that our certificates yield GPs which efficiently compute lower bounds both for unconstrained and constrained polynomial optimization problems. Particularly, our approach is significantly faster and often provides better bounds than semidefinite programming,

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