Fractional Turan theorems

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Abstract

Let $\mathfrak{G}$ be a family of graphs closed under induced subgraphs. We say that $\mathfrak{G}$ satisfies Turan property if given $\beta \in (0,1)$ there is $\alpha \in (0,1)$ (depending only on $\mathfrak{G}$ and β) such that if $G \in \mathfrak{G}$ and $|E(G)| \geq \beta\binom{|V(G)|}{2}$, then there is a clique of G of size bigger than $\alpha|V(G)|$.

Furthermore, if $\alpha = \frac{\beta}{K}$ for some constant $K > 0$, then we say that $\mathfrak{G}$ satisfies the linearly fractional Turan inequality.

For example, if $\mathfrak{G}$ is the family of intersection interval graphs, the fractional Helly theorem precisely states that $\mathfrak{G}$ satisfies the linearly fractional Turan inequality.

Let $\mathfrak{G}$ be a family of graphs closed under induced subgraphs. The Turan numbers for the family $\mathfrak{G}$ are defined as follows:

$$T(\mathfrak{G}, k, n) = \text{maximal number of edges of a graph in } \mathfrak{G}, \text{ with } n \text{ vertices, without a complete graph } K_{k+1}.$$

In this talk we present the following theorem.

Theotem. Let $\mathfrak{G}$ be a family of graphs closed under induced subgraphs. Then the following is equivalent.

a) The family $\mathfrak{G}$ satisfies the linearly fractional Turan property,

b) there is a constant K such that $T(\mathfrak{G}, k, n) < Knk$,

c) there is a constant $q > 0$ such that for every $G \in \mathfrak{G}$, we have that $\chi(G) \leq q\omega(G)$, and

there is a constant $\lambda > 0$ such that for every bipartite graph $B \in \mathfrak{G}$

$$|E(B)| \leq \lambda|V(B)|,$$

d) there is a constant c such that for every $G \in \mathfrak{G}$, $d(G) < c\omega(G)$.

This result includes many interesting families of graphs closed under induced subgraphs like for example the class of split graphs, interval graphs, claw free graphs, chordal graphs, etc In this talk, we shall discuss also the fractional Turan behavior of the class of intersection graphs of boxes in $\mathbb{R}^d$.

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