Optimal codes in projective spaces

Gregory Minton (Massachusetts Institute of Technology)

Abstract

It is known that, in projective spaces, a regular simplex which is also a 1-design is an optimal code. In fact, such a simplex is universally optimal. However, as the space and the number of points vary, these codes do not always exist. In real and complex projective spaces they seem to only exist in special cases, but in projective spaces over the quaternions and octonions their existence seems to be more common. In fact, simple dimension-counting arguments suggest that they should exist for a range of cardinalities, and moreover they should exist in positive-dimensional families. We show that these arguments can be formalized by computer-assisted proof; these proofs demonstrate rigorously that a true solution exists in a small neighborhood of a given approximate solution. The crux of our method is finding a minimal set of equalities determining a regular simplicial 1-design. Using this approach we prove the existence of a laundry list of new universally optimal codes, including tight 2-designs in the quaternionic and octonionic projective planes.