Probabilities of Optimal Networks Structures

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Abstract

Steiner problem of finding a shortest tree connecting a given finite subset M of a metric space is known as *NP*-complete, see [1], and the main reason is an exponentially large number of possible structures of the trees which can connect the terminal set M and possible additional vertices-forks. Similar difficulties appear in other one-dimensional geometrical variational problems, such as minimal fillings problem [2], etc.

We suggest to investigate probabilistic characteristics of possible combinatorial structures of optimal networks, i.e. we want to find out which structures appear quite often, and which — rather rarely.

As a main example we take minimal fillings of finite additive metric spaces, see definitions below. For that case a probabilistic model is constructed. Under this model, for a pair of binary trees with n vertices of degree 1 a formula characterizing the ratio of probabilities of these trees appearance as minimal filling structures is obtained [4]. An asymptotic behavior of the probabilities under $n \to \infty$ is investigated. The results are obtained in collaboration with V. N. Sal'nikov.

Necessary definitions. A connected weighted graph $\big(G = (V, E), \omega\big)$ with vertex set V, edge set E, and non-negative weight function $\omega\colon E \to \mathbb{R}$ is a *filling* of a finite metric space (M, ρ), see [2], if $M \subset V$ and for any points x, y from M the inequality $\rho(x, y) \le d_\omega(x, y)$ is valid, where d_ω is a distance function generated by ω on M. Namely, $d_\omega(x, y)$ is equal to the least possible weight of the pass connecting x and y in G. A filling with the least possible weight is referred as *minimal filling*. Also recall, that a finite metric space (M, ρ) is said to be *additive*, if its distance function ρ can be generated by a *generating tree*, a weighted tree (T, ω), whose degree 1 vertex set coincides with M, and such that $\rho(x, y) = d_\omega(x, y)$ for any $x, y \in M$, see [3]. In [2] it is proved that minimal fillings of finite additive metric spaces are exactly their generating trees.

References

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