

Commensurate Vectors, Commensurate Triangles and the Proliferation of Combinatorial Types of Parallelohedra

Robert Erdahl (Queen's University, Canada)

Abstract

One of the more striking, indeed daunting discoveries in the theory of parallelohedra is the astounding rate of growth of combinatorial types with dimension. For example, by restricting attention to the primitive types where the dual cell corresponding to each vertex is a simplex, this growth has been reported by Peter Engel to be:

Dimension	2	3	4	5	6
No. of types	1	1	3	222	185×10^6

The data available for the general case is much more forbidding. An important problem in the theory of parallelohedra is to gain some insight into how such rapid growth is possible.

In my talk I will describe a few simple construction that explain in part how rapid growth can occur in higher dimension, and even be expected. The constructions have as starting point the simple notions of commensurate lattice vector and commensurate lattice triangle. A commensurate lattice vector in the ambient lattice is one that can be translated so that it fits inside the parallelohedron, and similarly, a commensurate lattice triangle is one that can be translated so that it fits inside the parallelohedron.