Inner Spanning Trees Geometry for Planar Polygons

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A minimal spanning tree for a finite subset M of a metric space is a shortest connected graph with the vertex set M. Minimal spanning trees are used in applications as solutions to the shortest connection problem, since there exist polynomial algorithms constructing them. For $M \subset \mathbb{R}^2$, quadratic algorithms are known which are based on the concept of Delaunay triangulation of the set M. The Delaunay triangulations are of their own interest, see, for example [1].

A spanning tree for a vertex set M of a planar polygon is said to be *inner* if all its edges–segments lie in the polygon. An inner spanning tree of the least possible length is referred as *inner minimal spanning tree*. Such threes also appears in applications as solutions to optimal connection problems with obstacles. Besides, inner spanning trees are important in calculation of the Steiner ratio of the Euclidean plane and some other metric spaces, see [2] and [3]. We investigate geometrical properties of inner minimal spanning trees as minimal spanning trees in the planar polygon endowed with the inner metric. Analogues of Voronoi diagram and Delaunay triangulation are constructed [4]. It is proved that each minimal inner spanning tree is a subgraph of the Dealauney graph. Also, possible structure of faces of the Delaunay graph is described. Some of the obtained results can be generalized to the case of immersed planar polygons in the sense of [5]. The results are obtained in collaboration with A. A. Tuzhilin.

References

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