

Voronoi conjecture on parallelohedra for new special case

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Definition 1. A parallelohedron is a d -dimensional polytope which can tile a d -dimensional space with translation copies.

It is clear that Dirichlet-Voronoi polytope for an arbitrary d -dimensional lattice Λ is parallelohedron. In 1908 Voronoi conjectured that every parallelohedron can be constructed by taking Dirichlet-Voronoi polytope for some lattice.

Conjecture 1 (Voronoi, 1908). Every parallelohedron is an affine image of Dirichlet-Voronoi polytope for some lattice.

Since Voronoi states his conjecture there were several results for different families of parallelohedra (G.Voronoi, O.Zhitomirskii, R.Erdahl, A.Ordine) but the conjecture remains unproved in general case.

In this talk we will discuss some of the mentioned results and the way they were achieved. Also we will sketch a proof of the Voronoi conjecture for a new special case.

Theorem 1. *The Voronoi conjecture is true for parallelohedron P if the surface of P remains simply connected after deletion of closed non-primitive faces of codimension 2.*

Also we will give a way to generalize this theorem and discuss some open problems in parallelohedra theory.

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