Minkowski Sums of Voronoi Polytopes and Commensurate Delone Tilings

Robert Erdahl (Queen's University, Canada)

Abstract

It is natural to ask whether the Voronoi polytope for a lattice can be written as the Minkowski sum of polytopes that are also Voronoi polytopes for lattices. S. S. Ryshkov answered this question by showing that the Voronoi polytope for a point in the relative interior of an L-type is affinely equivalent to a weighted Minkowski sum of Voronoi polytopes for the edge forms of the L-type.

I will initiate proceedings by sketching the proof of the Lemma: A Voronoi polytope can be written as the Minkowski sum of Voronoi polytopes if and only if the Delone tilings for the two summands are commensurate (a notion that will be defined during the lecture). This Lemma serves as a corner stone for a beautiful duality theory that relates commensurate Delone tilings and the Minkowski decomposition of Voronoi polytopes; it also provides the key step in proving Ryshkov's Theorem. The line of argument I use will parallel that used in a preliminary version of the duality theory relating dicings and Voronoi zonotopes.