Packing equal disks on a triangular lattice torus

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Suppose we have a periodic packing in the plane of equal disks such that the period lattice is the usual triangular lattice. When you take the quotient of this packing by the lattice, the infinite packing becomes a finite packing of n equal disks in a flat torus, which we call the *triangular torus*. When the infinite packing is the close packing, where each disk is surrounded by 6 others, it achieves the maximal possible density $\pi/\sqrt{12}$. Suppose one takes a sublattice of the triangular lattice, which is itself a triangular lattice. The quotient corresponds to a packing of n disks in a triangular torus whose density is $\pi/\sqrt{12}$, the maximal. But this only happens when $n = a^2 + ab + b^2$, for a, b integers, which we call a *triangle lattice number*. We conjecture that the maximal density of n disks in a triangular torus, when n is not a triangular lattice number, is at most $\frac{n}{n+1}\pi/\sqrt{12}$, and show that it is true for some special cases for the contact graph of the packing.