Schedule of talks.

Wednesday (April 18)
Afternoon session

2:00-2:45

Alexander Ivanov (Moscow State University)

Inner Spanning Trees Geometry for Planar Polygons

Abstract. A minimal spanning tree for a finite subset M of a metric space is a shortest connected graph with the vertex set M. Minimal spanning trees are used in applications as solutions to the shortest connection problem, since there exist polynomial algorithms constructing them. For $M \subset \mathbb{R}^2$, quadratic algorithms are known which are based on the concept of Delaunay triangulation of the set M. The Delaunay triangulations are of their own interest, see, for example [1].

A spanning tree for a vertex set M of a planar polygon is said to be *inner* if all its edges–segments lie in the polygon. An inner spanning tree of the least possible length is referred as *inner minimal spanning tree.* Such threes also appears in applications as solutions to optimal connection problems with obstacles. Besides, inner spanning trees are important in calculation of the Steiner ratio of the Euclidean plane and some other metric spaces, see [2] and [3]. We investigate geometrical properties of inner minimal spanning trees as minimal spanning trees in the planar polygon endowed with the inner metric. Analogues of Voronoi diagram and Delaunay triangulation are constructed [4]. It is proved that each minimal inner spanning tree is a subgraph of the Dealauney graph. Also, possible structure of faces of the Delaunay graph is described. Some of the obtained results can be generalized to the case of immersed planar polygons in the sense of [5]. The results are obtained in collaboration with A. A. Tuzhilin.

References

[1] F. Preparata, M. Shamos *Computational Geometry: An Introduction*, Berlin, Heidelberg, Springer-Verlag (1990).

[2] D. Z. Du, F. K. Hwang, *A proof of Gilbert–Pollak Conjecture on the Steiner ratio*, Algorithmica, **7**, pp. 121–135, 1992.

[3] A. O. Ivanov, A. A. Tuzhilin, *Steiner Ratio. The State of the Art*, Matemat. Voprosy Kibern., **11**, pp. 27–48, 2002.

[4] A. O. Ivanov, A. A. Tuzhilin, *Inner Spanning Trees Geometry for Planar Polygons*, Izvestiya RAN, Ser. matem.,**76**, No. 2, pp. 3-36, 2012.

[5] A. O. Ivanov, A. A. Tuzhilin, *Immersed polygons and their diagonal triangulations* Izv. Math., **72**, No. 1, pp. 63-90, 2008.

Wednesday (April 18)
Afternoon session

3:00-3:45

Alexey Tuzhilin (Moscow State University)

Stabilization theorems for minimal networks.

Abstract.

We discuss one method enabling to construct various examples of Steiner Minimal Trees (SMT) possessing predetermined properties. It is well-known that each SMT in the Euclidean plane consists of straight segments meeting each other at their ending points by the angles of at least 120°. We call the trees of this local structure by *local minimal trees* (LMT). Not each LMT is an SMT. Our stabilization theorem states that on any LMT one can add sufficiently dense finite set of boundary points of degree 2 to convert the tree into SMT (this is joint result with A.O.Ivanov). The stabilization theorem was used to obtain a complete description of all possible germs of closed lunes for edges of an SMT (joint result with A.O.Ivanov and O.A.S'edina). Stabilization theorem was generalized to the class of shortest forests with partially free boundary. Namely, we consider a collection of nonintersecting LMTs as the boundary set and join them by a forest to obtain a connected graph; we minimize the lengths of such forests and assume that for each the shortest forest its union G with the boundary LMTs is a LMT. Then we prove that one can provide the boundary LMTs with a sufficiently dense finite set of boundary points of degree 2 to convert G into SMT (joint result with A.O.Ivanov and A.E.Mel'nikova).

Wednesday (April 18)
Afternoon session

4:00-4:45

Alexey Garber (Moscow State University; Delone Lab, Yaroslavl)

with Andrey Gavrilyuk (Steklov Mathematical Institute, Moscow; Delone Lab, Yaroslavl) and Alexander Magazinov(Steklov Mathematical Institute, Moscow; Delone Lab, Yaroslavl)

Voronoi conjecture on parallelohedra for new special case

Abstract.

Definition 1. A parallelohedron is a d-dimensional polytope which can tile a d-dimensional space with translation copies.

It is clear that Dirichlet-Voronoi polytope for an arbitrary d-dimensional lattice Λ is parallelohedron. In 1908 Voronoi conjectured that every parallelohedron can be constructed by taking Dirichlet-Voronoi polytope for some lattice.

Conjecture 1 (Voronoi, 1908)**.** Every parallelohedron is an affine image of Dirichlet-Voronoi polytope for some lattice.

Since Voronoi states his conjecture there were several results for different families of parallelohedra (G.Voronoi, O.Zhitomirskii, R.Erdahl, A.Ordine) but the conjecture remains unproved in general case.

In this talk we will discuss some of the mentioned results and the way they were achieved. Also we will sketch a proof of the Voronoi conjecture for a new special case.

Theorem 1. *The Voronoi conjecture is true for parallelohedron P if the surface of P remains simply connected after deletion of closed non-primitive faces of codimnesion* 2*.*

Also we will give a way to generalize this theorem and discuss some open problems in parallelohedra theory.

Thursday (April 19)
Morning session

9:45-10:30

Robert Connelly (Cornell University)

with Will Dickinson (Grand Valley State University)

Packing equal disks on a triangular lattice torus

Abstract. Suppose we have a periodic packing in the plane of equal disks such that the period lattice is the usual triangular lattice. When you take the quotient of this packing by the lattice, the infinite packing becomes a finite packing of n equal disks in a flat torus, which we call the *triangular torus.* When the infinite packing is the close packing, where each disk is surrounded by 6 others, it achieves the maximal possible density $\pi/\sqrt{12}$. Suppose one takes a sublattice of the triangular lattice, which is itself a triangular lattice. The quotient corresponds to a packing of n disks in a triangular torus whose density is $\pi/\sqrt{12}$, the maximal. But this only happens when $n = a^2 + ab + b^2$, for a, b integers, which we call a *triangle lattice number.* We conjecture that the maximal density of n disks in a triangular torus, when n is not a triangular lattice number, is at most $\frac{n}{n+1}\pi/\sqrt{12}$, and show that it is true for some special cases for the contact graph of the packing.

Thursday (April 19)
Morning session

10:40-11:25

Nikolai Dolbilin (Steklov Mathematical Institute, Moscow; Delone Lab, Yaroslavl)

Crystalline Structure: Local and Global Approaches

Abstract. An appropriate concept for describing an arbitrary discrete atomic structure is the Delone set (or an (r, R)-system). Structures with long-range order such as crystals involve the concept of space group as well.

A mathematical model of an ideal monocrystalline matter is defined now as a Delone set which is invariant with respect to some space group. One should emphasize that under this definition the well-known periodicity of a crystal in all 3 dimensions is not an additional requirement, because full periodicity is implied by the celebrated Schoenflies-Bieberbach theorem, any space group contains a translational subgroup with a finite index.

Thus, a mathematical model of an ideal crystal uses two concepts: a Delone set (which is of local character) and a space group (which is of global character).

Since crystallization is a process which results from mutual interaction of just nearby atoms, it is believed (L. Pauling, R. Feynmann et al) that the long-range order of atomic structures of crystals (and quasi-crystals too) emerges from local rules restricting the arrangement of nearby atoms.

However, there were no whatever rigorous results in this directon until the 1970s, when Delone and his students initiated a local theory of crystals. The main aim of this theory was (and is) the rigorous derivation of the space group symmetry of a crystalline structure from the pair-wise identity of local arrangements around each atoms.

In the talk I will present results on local rules for crystals rules and also a quite opposite approach to the description of crystal.

Keywords: global order, space group, local rule, Delone set, crystal

Thursday (April 19)
Morning session

11:35-12:20

Robert Erdahl (Queens University, Canada)

Minkowski Sums of Voronoi Polytopes and Commensurate Delone Tilings

Abstract. It is natural to ask whether the Voronoi polytope for a lattice can be written as the Minkowski sum of polytopes that are also Voronoi polytopes for lattices. S. S. Ryshkov answered this question by showing that the Voronoi polytope for a point in the relative interior of an L-type is affinely equivalent to a weighted Minkowski sum of Voronoi polytopes for the edge forms of the L-type.

I will initiate proceedings by sketching the proof of the Lemma: A Voronoi polytope can be written as the Minkowski sum of Voronoi polytopes if and only if the Delone tilings for the two summands are commensurate (a notion that will be defined during the lecture). This Lemma serves as a corner stone for a beautiful duality theory that relates commensurate Delone tilings and the Minkowski decomposition of Voronoi polytopes; it also provides the key step in proving Ryshkovs Theorem. The line of argument I use will parallel that used in a preliminary version of the duality theory relating dicings and Voronoi zonotopes.

Thursday (April 19)
Afternoon session

2:00-2:45

Alexander Dranishnikov (University of Florida)

On dimension growth of discrete groups

Abstract. We define the notion of dimension growth of infinite graphs in terms of coloring of vertices. We apply it to the Cayley graphs of finitely generated groups and make some computations. Then we show how some famous conjectures in Topology and Algebra can be reduced to an estimate of the dimension growth of a group. This is a joint work with M. Sapir.

Thursday (April 19)
Afternoon session

3:00-3:45

Andras Bezdek (Auburn University)

On fair triangulations of polygons and polyhedra

Abstract. A convex partition of a polygon P is a finite set of convex polygons such that the interiors of the polygons do not intersect and the union of the polygons is equal to the original polygon P. The desire to create optimal partitions of a given convex polygon furnished a number of problems in discrete geometry. The properties used in optimization among others include equal area, equal perimeter and the number of pieces. The concept of fair partitions commonly refers to problems where simultaneously several properties need to be optimized. Variations of the cake-cutting problem are the most known problems among these. This talk surveys some of the $2D$ and $3D$ results and introduces some new variants. We are particularly interested in optimization problems which are restricted to triangulations only.

Thursday (April 19)
Afternoon session

4:00-4:45

Wlodzimierz Kuperberg (Auburn University)

with Andras Bezdek (Auburn University)

Packing cones in space

Abstract. One of the basic problems in discrete geometry is to determine the most efficient (densest) packing of congruent replicas of a given convex body in the plane or in space. Several types of the problem arise depending on the kind of isometries allowed for the packing: packing by translations, lattice packing, translations and point reflections, or all isometries. The problem of dense packing of $\mathbb{R}^3$ with cones over a convex disk is the subject of our interest, especially for packing by translations or translations and point-reflections. We state several open problems and present some density bounds.

Friday (April 20)
Morning session

9:30-10:15

Frank Vallentin (TU Delft, Netherlands)

Convex formulations and relaxations of geometric packing problems

Abstract. Packing problems in geometry can be formulated as finding the weighted independence number of an infinite graph. A classical result due to Motzkin and Strauss gives a characterization of the independence number of a finite graph in terms of copositive optimization. In this talk I will extend this result to infinite graphs. For this a duality theory between the primal cone of copositive kernels and the dual cone of completely positive measures is developed.

One way to relax this copositive formulation is to use semidefinite optimization and the weighted theta number of Grötschel, Lovász, Schrijver. I will demonstrate this in the case of multiple-size sphere packings. To approximate this infinite dimensional semidefinite program the use of tools from polynomial optimization are essential. Next to presenting numerical results (complementing recent results by Hopkins, Jiao, Stillinger, Torquato) I will emphasize the importance of the right choice of polynomial basis functions to guarantee numerical stability.

(based on joint works with David de Laat, Fernando Mario de Oliveira Filho, Cristian Dobre, Mirjam Dür)

Friday (April 20)
Morning session

10:25-11:10

Abhinav Kumar (Massachusetts Institute of Technology)

Rigidity of spherical codes, and kissing numbers

Abstract. I will report on joint work with Cohn, Jiao and Torquato, in which we studied rigidity or jamming properties of spherical codes. In particular, we described a linear programming algorithm to detect whether a code is infinitesimally rigid (i.e. cannot be deformed without decreasing the minimal distance), and several examples coming from kissing configurations in low dimensions. We used a variant of these techniques to improve the kissing numbers in dimensions 25 through 31.

Friday (April 20)
Morning session

11:20-11:45

Wei-Hsuan Yu (University of Maryland)

New bounds for spherical two-distance sets

Abstract. A spherical two-distance set is a finite collection of unit vectors in $\mathbb{R}^n$ such that the set of distances between any two distinct vectors has cardinality two. We use the semidefinite programming method to compute improved estimates of the maximum size of spherical two-distance sets. Exact answers are found for dimensions $n = 23$ and $40 \leq n \leq 93$ (except 46, 78), where previous results gave divergent bounds.

Friday (April 20)
Morning session

11:55-12:20

Andrey Nikolaev (Yaroslavl State University)

with Vladimir Bondarenko(Yaroslavl State University)

On the vertices of the cut polytope relaxations and associated hypergraphs

Abstract. We study a relationship between a special class of hypergraphs and properties of the relaxation points $M_{n,k}$ of a cut polytope. We prove that, for sufficiently large n, the polytopes $M_{n,4}$ and $M_{n,5}$ have points whose decompositions in the vertices of $M_{n,3}$ contain no integer vertices.

Friday (April 20)
Afternoon session

2:00-2:30

Alexander Barg (University of Maryland)

Effective version of Shannon's theorem in information theory

Abstract. According to Shannon's capacity theorem, information can be reliably transmitted over noisy channels as long as the rate of transmission does not exceed the channel capacity. The original proof (1948) involves the probabilistic method. The first effective version of this theorem was found in 2008 by E. Arikan. We discuss Arikan's idea as well as an extension of his results due to Woomyoung Park and the speaker.

Friday (April 20)
Afternoon session

2:45-3:30

Igor Pak (University of California, Los Angeles)

Geometric realization of convex polyhedra and polyhedral complexes

Abstract. Given a polyhedral complex embeddable in $\mathbb{R}^3$, is it always possible to realize it geometrically? If yes, can one always make coordinates integer? If yes, how large integers does one need? I will survey what is known on all 3 questions, present our recent results in all three directions, and state few conjectures. Joint work with Stedman Wilson.

Friday (April 20)
Afternoon session

3:45-4:30

Michael Boshernitzan (Rice University)

Local complexity of finite sets

Abstract. By *local complexity* $\phi(X)$ of a finite subset $X \subset [0,1) = S^1$ of the unit circle we mean the cardinality of the set of distinct lengths of the intervals in the partition (of S^1) determined by X. We discuss various generalizations, old and new, of the 3-distance theorem claiming that

$$\phi\big(\{\langle k\alpha\rangle \mid 1 \le k \le n\}\big) \le 3,$$

for all real α and $n \ge 1$. (Here $\langle x\rangle$ stands for the fractional part of x).

In other words, the 3-distance theorem claims that the finite pieces of orbits of rotations $R_\alpha \colon S^1 \to S^1$ have a uniform bound (namely, 3) on their local complexity. The result extends from rotations R_α to a larger class of transformations, in particular to all interval exchange transformations. (These transformations form a natural generalization of rotations and provide important examples in ergodic theory).

We review other examples when large sets with small local complexity appear. In particular [F. Dyson, 1991], for $\alpha = \sqrt[3]{2}$ and $\beta = \sqrt[3]{4}$,

$$\sup_{n\ge 1} \phi\Big(\Big\{\langle m\alpha + n\beta\rangle \Big| \ 1 \le m, n \le n\Big\}\Big) < \infty.$$

The above theorem extends for badly approximable pairs (α, β) in Schmidt's sense (but not to all pairs), see recent preprint by P. Bleher, R. Roeder, Y. Homma, L. Ji, and J. Shen (Arxiv 2011).

A number of open questions will be posed. (In particular, what is the right definition of the multidimensional local complexity?).

Friday (April 20)
Afternoon session

4:40-5:05

Andrey Gavrilyuk (Steklov Mathematical Institute, Moscow)

Lifting tiles around a face

Abstract. An old problem is to determine whether a given convex tiling C^d of $\mathbb{R}^d$ is a projection of some convex polyhedron P in $\mathbb{R}^{d+1}$ (C. Davis, F. Aurenhammer, P. McMullen). There is a number of useful and handy criteria of being such projection. Most of the criteria deal with global structure while usually we can construct just local objects (expecting them to join together later). We provide a new local criterion which could be of use in cases when we know just local structure well. This criterion connects (a bit unexpectedly) the topic with a topic of polytopal fans.

Saturday (April 21)
Morning session

9:30-10:15

Elena Poletaeva (University of Texas – Pan American)

Finite W-algebras for Lie algebras and superalgebras

Abstract. The finite W-algebras are certain associative algebras associated to a complex semisimple Lie algebra $\mathfrak{g}$ and a nilpotent element e of $\mathfrak{g}$. A finite W-algebra W_e is a generalization of the universal enveloping algebra $U(\mathfrak{g})$. For $e = 0$, W_e is simply $U(\mathfrak{g})$. It is a result of B. Kostant that for a regular nilpotent element e, W_e coincides with the center of $U(\mathfrak{g})$.

We shall present some basic constructions of finite W-algebras, and show that certain results about them can be generalized for classical simple Lie superalgebras.

Saturday (April 21)
Morning session

10:30-11:15

Bao-Feng Feng (University of Texas – Pan American)

Free fermionic approach to the KP and two-dimensional Toda-lattice hierarchies

Abstract. In this talk, a survey on the free fermionic approach to the KP and two-dimensional Toda-lattice (2DTL) hierarchies will be given based on the collective works by Date, Jimbo, Kashiwara, Miwa, Takasaki and Takebe. First, we will show how the bilinear identities, as well as their Gram-type solutions, are obtained by embedding the infinite dimensional Lie algebras in the context of free fermions. Then, we will also show both the continuous and discrete soliton equations such as the KdV and the mKdV equations, Hirota-Miwa equation, and fully discrete analogue to the 2DTL lattice equations can be obtained from these generic bilinear identities.

Saturday (April 21)
Morning session

11:30-12:15

Tim Huber (University of Texas – Pan American)

Combinatorial applications of a fundamental set quintic of relations

Abstract. In this lecture we apply properties of four quintic theta functions paralleling those of the classical Jacobi null theta functions. The quintic theta functions satisfy analogues of Jacobi's quartic theta-function identity and counterparts of Jacobi's Principles of Duplication and Dimidiation. The resulting library of quintic transformation formulas will be used to describe the action of Hecke operators of level five and more general quintic dissection operators. This machinery will be used to obtain interesting quintic multisections for Eisenstein series and related generating functions. Among the many consequences are Ramanujan's expansions for quintic dissections of the partition function and generalizations thereof. Central to our analysis is a new nonlinear coupled system of differential equations satisfied by the quintic theta functions.