

GAUSS CIRCLE PROBLEM OVER SMOOTH INTEGERS

Speaker: Ankush Goswami

Abstract:

A classical problem in mathematics, the *Gauss circle problem* is to determine how many integer lattice points are inside a circle of radius \sqrt{x} ($x > 0$) centered at the origin. In other words, it is the estimate of the number of points in $N(x) := \{(a, b) \in \mathbb{Z}^2 : a^2 + b^2 \leq x\}$, and Gauss showed that $|N(x)| \sim \pi x$, so the real problem is to accurately bound the error term describing how the count of (lattice) points differs from the area of the circle. In this talk, we consider a variant of the circle problem wherein we consider only those lattice points $(a, b) \in N(x)$ such that $a^2 + b^2$ is y -smooth where $0 < y \leq x$. If we denote the set of such points by $N(x, y)$, then we show that $|N(x, y)| \sim \rho(\alpha)\pi x$ where $\alpha = \log x / \log y$, and $\rho(\alpha)$ is the Dickman function satisfying a delay-differential equation. We will show that this result is uniform in y over a certain large range. If time permits, I will discuss a generalization of the circle problem by introducing a new function and obtain certain estimates following a method of Selberg.

Time: 1:30-2:30 pm, November 3, 2023

Location: BLH5B 1.316 and in

Zoom: <https://utrgv.zoom.us/j/83585846705>



Cookies and Coffee will be provided!