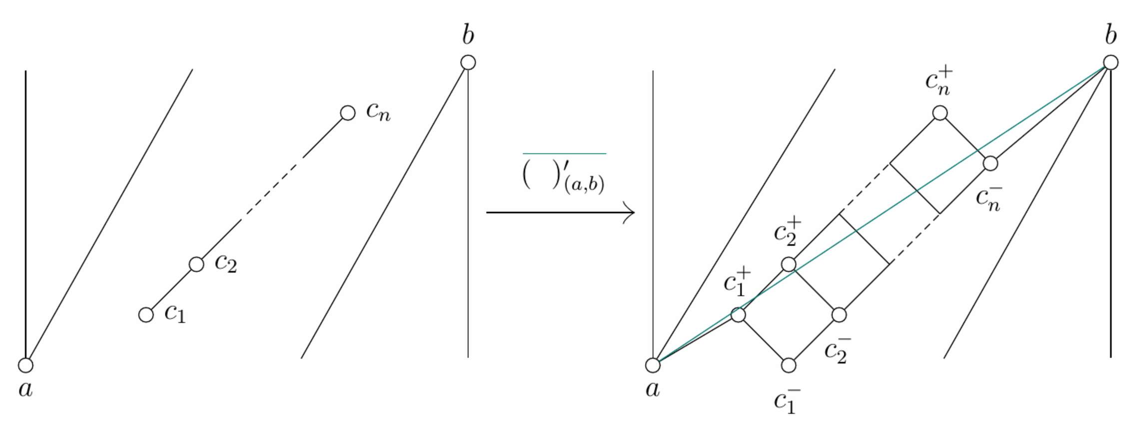
Matrix Representation of Differentiation Algorithm with respect to a suitable pair.



Maria del Rosario Valencia Arévalo

Coauthor: Harrinson Andres Arrubla Ramirez

University of Texas Rio Grande Valley and Universidad Nacional de Colombia

mariadelrosario.valenciaarevalo01@utrgv.edu and harrinson.arrubla01@utrgv.edu

Abstract

The representation of partially ordered sets (posets) over an arbitrary field were introduced by Nazarova and Roiter, together with a differentiation algorithm to classify the posets representations of width less than or equal to 3. These representations are presented in matrices, transforming a partially ordered sets representation problem into a matrix problem. In this research we present in detail the matrix version of the differentiation algorithm with respect to suitable pair of points.

Differentiation with respect to suitable pair

A pair (a,b) is suitable if $\mathscr{P} = a^{\triangledown} + b_{\vartriangle} + C$, with $C = \{c_1 < \cdots < c_n\}$ a (possibly empty) chain. For such a pair, we let

$$\mathscr{P}'_{(a,b)} = \left((a^{\triangledown} + b_{\vartriangle})\backslash C\right) \cup C^+ \cup C^-.$$

We have a differentiation functor

$$(\ \ )'_{(a,b)} \colon \operatorname{rep}(\mathscr{P},k) \to \operatorname{rep}(\mathscr{P}'_{(a,b)},k)$$

which assigns to each object $U = (U_0; U_x \mid x \in \mathscr{P})$ the derived object $U'_{(a,b)} = U' = (U'_0; U'_x \mid x \in \mathscr{P}'_{(a,b)})$ given by

$$U'_0 = U_0$$
$$U'_{c_i^+} = U_a + U_{c_i} \text{ and } U'_{c_i^-} = U_b \cap U_{c_i}, \text{ for all } i$$
$$U'_x = U_x, \text{ for all others points.}$$

If $\phi \colon U \to V$ is a morphism in $\operatorname{rep}(\mathscr{P},k)$ we define $\phi'_{(a,b)} = \phi' = \phi$.

Theorem:
With the same notations as above, the differentiation functor $(\ \ )'_{(a,b)}$ induces a quotient categories equivalence

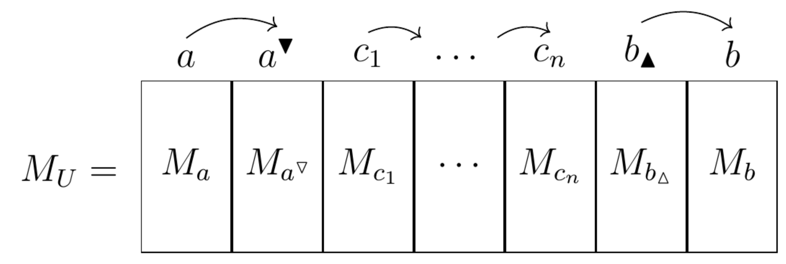
$$\operatorname{rep}(\mathscr{P},k)/\langle\{k(a), k(a,c_1), \dots, k(a,c_n)\}\rangle \rightleftarrows \operatorname{rep}(\mathscr{P}'_{(a,b)},k)/\langle k(a)\rangle.$$

In particular,

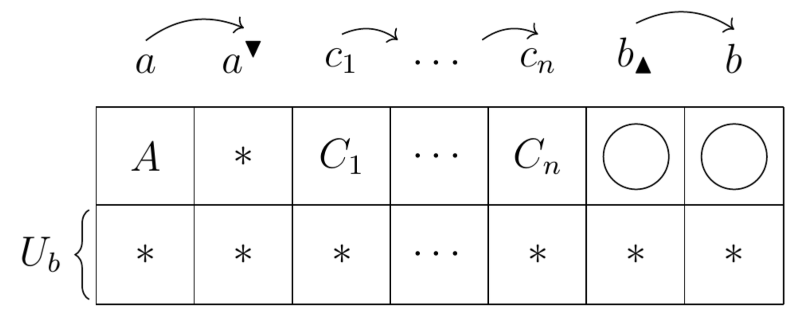
$$|\operatorname{Ind}(\mathscr{P},k)| = |\operatorname{Ind}(\mathscr{P}'_{(a,b)},k)| + n.$$

Matrix Representation

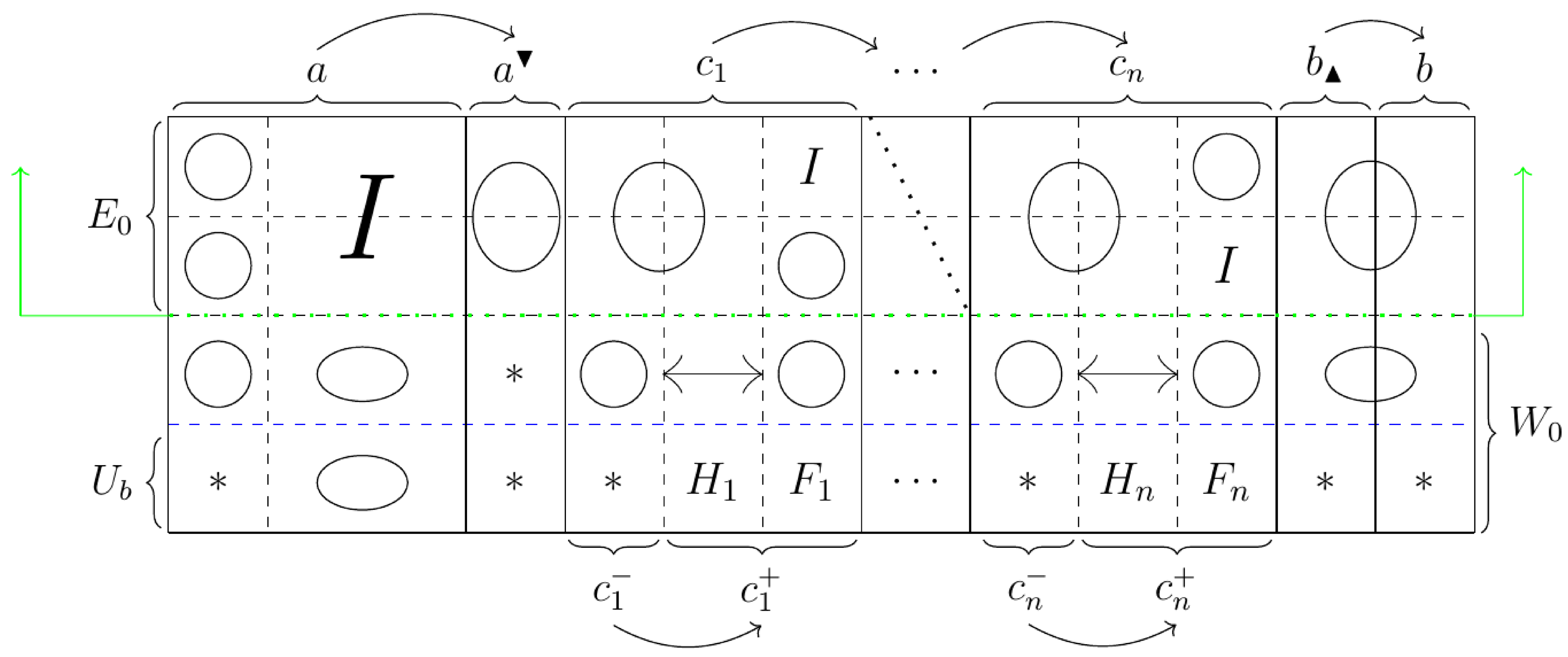
Let $(\mathscr{P},k)$ be a poset with a suitable pair, and $\omega(\mathscr{P}) \leq 3$. Let U in $\operatorname{rep}(\mathscr{P},k)$ and M_U a matrix presentation of U.



The lines are the order relation between the points. In other words the operations between columns in that direction. Then we split the rows using U_b.



Using admitted transformations on the matrix. With (E_0, W_0) be the complementary pair of (U_a, U_b).



$\leftrightarrow$ linearly independent columns, I identity, $*$ an arbitrary matrix and H_n, F_n matrix for c_n^+.

Integration

Let $W^{\uparrow} \in \operatorname{rep}(\mathscr{P},k)$ such that $(W^{\uparrow})' = W \oplus k^m(a)$. The integration is not a functorial operation. Define $Z_{c_j^+} = F_j \oplus H_j \oplus \underline{Z}_{c_j^+}$ and $Z_b \cap Z_{c_j^+} = F_j \oplus \left(\sum_{x<c_j^+} Z_b \cap Z_x\right)$.

Let $d_j = \dim F_j$ and fix $\{f_1^j, \dots, f_{d_j}^j\}$ a base for F_j over k. Let E a k-space with $\dim E = m = \sum_{j=1}^{n} d_j$ with a fix base $\{e_i^j \mid i \in \{1,\dots,d_j\}\}$.Let $g_i^j = e_i^j + f_i^j \in Z_0^{\uparrow}$. We Define $Z^{\uparrow}_{(a,b)} = Y = (Y_0, Y_x \mid x \in \mathscr{P})$ in $\operatorname{rep}(\mathscr{P},k)$ as follows:

$$Y_0 = E \oplus Z_0,$$
$$Y_x = E \oplus Z_x \quad \text{si } x \in a^{\triangledown},$$
$$Y_x = E \oplus Z_x \quad \text{si } x \in b_{\vartriangle},$$
$$Y_{c_1} = H_1 + Z_{c_1^-} + kg_1^1 + \cdots + kg_{d_1}^1,$$
$$Y_{c_j} = H_j + Z_{c_j^-} + Z_{c_{j-1}^-} + kg_1^j + \cdots + kg_{d_j}^j \qquad \text{for } j \in \{2,\dots,n\},$$

References

- Nazarova, L.A. and A.V Roiter(1972). «Representations of partially ordered sets». Zap. Nauchn. Sem. LO-MI 28, pag. 5-31.
- Medina, G. and A.G Zavadskij (2004) «The fur sub space problem: An elementary solution». inear Algebra and its Applications 392, pag 11-23
- Zavadskij , A. (2005) «On Two-Point Differentiation and its Generalization». AMS
- Nazarova, L. A. (1967). «Representations of a tetrad». Izv. Akad Nauk SSSR, Ser.Matem. 31, No. 6, pag 1361-1378.
- Zavadskij, A. (1977). «Differentiation with respect to a pair of points.» Russian. Matrix Problems. Ed. by Y. A. Mitropolskii y L. A. Nazarova, pag. 115-121.