The weighted Singer conjecture for Coxeter groups in dimensions three and four

Wiktor J. Mogilski
(Univeristy of Wisconsin – Milwaukee)

Abstract

Associated to a Coxeter system (W, S) there is a contractible simplicial complex Σ called the Davis complex on which W acts properly and cocompactly by reflections. Given a positive real multiparameter $\mathbf{q}$, one can define the weighted L^2-(co)homology groups of Σ and associate to them a nonnegative real number called the weighted L^2-Betti number. Within the spectrum of weighted L^2-(co)homology, there is a conjecture of interest called the Weighted Singer Conjecture which was formulated in a 2007 paper of Davis–Dymara–Januszkiewicz–Okun. The conjecture claims that if Σ is an n-manifold (equivalently, the nerve of the corresponding Coxeter group is an $(n-1)$-sphere), then the weighted L^2-(co)homology groups of Σ vanish above dimension $\frac{n}{2}$ whenever $\mathbf{q} \leq \mathbf{1}$ (that is, all terms of the multiparameter $\mathbf{q}$ are real numbers less than or equal to 1). We present a proof of the conjecture in dimension three that encompasses all but nine Coxeter groups. Then, under some restrictions on the nerve of the Coxeter group, we obtain partial results whenever $n = 4$ (in particular, the conjecture holds for $n = 4$ if the nerve of the corresponding Coxeter group is a flag complex). We then extend our results in dimension four to prove a general version of the conjecture for the case where the nerve of the Coxeter group assumed to be a flag triangulation of a 3-manifold.