Complex geometry and computational complexity of the permanent

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Abstract

The permanent of a square matrix is defined almost like the determinant, only simpler: all monomials are counted with the "+" sign. Permanents of 0-1 matrices are of interest to combinatorics as they enumerate perfect matchings in bipartite graphs and permanents of complex matrices are of interest in physics as they "enumerate" bosons. We prove that the permanent of an nxn complex matrix is never 0 provided all the entries of the matrix are within distance 0.275 from 1 (the exact value of the constant is not known, though one cannot replace 0.275 by 0.710, say). Consequently, the permanent of an nxn complex matrix can be computed within a relative error epsilon in quasi-polynomial $n^{O(\ln n - \ln \epsilon)}$ time provided all the matrix entries are within distance 0.274 from 1. The method is readily extended to hafnians, multi-dimensional permanents and other partition functions.