

Circumcenter of Mass and the generalized Euler line

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Abstract

I shall define and study a variant of the center of mass of a polygon, called the Circumcenter of Mass. The Circumcenter of Mass is an affine combination of the circumcenters of the triangles in a non-degenerate triangulation of a polygon, weighted by their areas, and it does not depend on the triangulation. For an inscribed polygon, this center coincides with the circumcenter. The Circumcenter of Mass satisfies an analog of the Archimedes Lemma, similarly to the center of mass of the polygonal lamina. The line connecting the circumcenter and the centroid of a triangle is called the Euler line. Taking an affine combination of the circumcenters and the centroids of the triangles in a triangulation, one obtains the Euler line of a polygon. The construction of the Circumcenter of Mass extends to simplicial polytopes and to the spherical and hyperbolic geometries.